Galactic Nuclei as Collapsed Old Quasars

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Powerful emissions from the centres of nearby galaxies may represent dead quasars.

RYLE gives good evidence[1] that quasars evolve into powerful radio sources with two well separated radio components, one on each side of the dead or dying quasar. The energies involved in the total radio outbursts are calculated to be of the order of 10^{61} erg, and the optical variability of some quasars indicates that the outbursts probably originate in a volume no larger than the solar system. Now 10^{61} erg have a mass of 10^{40} g or nearly 10^7 Suns. If this were to come from the conversion of hydrogen into helium, it can only represent the nuclear binding energy, which is 3/400 of the mass of hydrogen involved. Hence 10^9 solar masses would be needed within a volume the size of the solar system, which we take to be 10^{15} cm (10 light h). But the gravitational binding energy of 10^9 solar masses within 10^{15} cm is GM^2/r which is 10^{62} erg. Thus we are wrong to neglect gravity as an equal if not a dominant source of energy. This was suggested by Fowler and Hoyle[2], who at once asked whether the red-shifts can also have a gravitational origin. Greenstein and Schmidt[3], however, earlier showed that this is unlikely because the differential red-shift would wash out the lines. Attempts to avoid this difficulty have looked unconvincing, so I shall adopt the cosmological origin for quasar red-shifts. Even with this hypothesis the numbers of quasar-like objects are very large, or rather they were so in the past. I shall assume that the quasars were common for an initial epoch lasting 10^9 yr, but that each one only remained bright for 10^6 yr, and take Sandage's estimate (quoted in ref. 4) of 10^7 quasar-like objects in the sky down to magnitude 22. This must represent a snapshot of the quasar era, so only one in a thousand would be bright. If these represent all the quasar-like objects that there are, then the density of dead ones should be $10^7 \times 10^3 = 10^{10}$ per Hubble volume. The distance between neighbouring dead ones is then an average of 10^{-3} Hubble distances (10^{10} light yr) or 3 Mpc. From these statistics it seems probable that a dead quasar-like object inhabits the local group of galaxies and we must expect many nearer than the Virgo cluster and $M87$. If we restrict ourselves to old quasars bright at radio wavelengths, then Sandage reduces his estimate by a factor of 200 so the average distance between dead quasars is around 20 Mpc. This is typical of the distance between clusters of galaxies.

If some 10^7–10^9 solar masses were involved in the quasar, releasing 10^7 solar masses as energy, then the dead quasar is likely still to be in the range 10^7–10^9 solar masses and to be bound still within a radius of the size of the solar system. Such an object is unlikely to exist for 10^{10} yr without burning out its nuclear fuel. There are no equilibria for burnt-out bodies of masses considerably in excess of a solar mass, however. Even uniform rotation hardly increases Chandrasekhar's critical mass of about $1 \cdot 4\ M_\odot$ and non-uniform rotation always leads to the generation of magnetic fields and to angular momentum transport. For masses already of the size of the solar system such periods of angular momentum transport will not be very long. In a few thousand years the outer parts acquire a large fraction of the angular momentum and slow down in circular orbit while the more massive inner portion contracts and spins faster. This central portion will collapse and finally fall within its Schwarzschild radius and be lost from view. Nothing can ever pass outwards through the Schwarzschild sphere of radius $r = 2\ GM/c^2$, which we shall call the Schwarzschild throat. We would be wrong to conclude that such massive objects in space-time should be unobservable, however. It is my thesis that we have been observing them indirectly for many years.

Effects of Collapsed Masses

As Schwarzschild throats are considerable centres of gravitation, we expect to find matter concentrated toward them. We therefore expect that the throats are to be found at the centres of massive aggregates of stars, and the centres of the nuclei of galaxies are the obvious choice. My first prediction is that when the light from the nucleus of a galaxy is predominantly starlight, the mass-to-light ratio of the nucleus should be anomalously large.

We may expect the collapsed bodies to have a broad spectrum of masses. True dead quasars may have 10^{10} or $10^{11}\ M_\odot$ while normal galaxies like ours may have only 10^7–$10^8\ M_\odot$ down their throats. A simple calculation shows that the last stable circular orbit has a diameter of $12\ GM/c^2 = 12m$ so we shall call the sphere of this diameter the Schwarzschild mouth. Simple calculations on circular orbits yield the following results, where M_7 is the mass of the collapsed body in units of $10^7\ M_\odot$, so that M_7 ranges from 1 to 10^4.

Circular velocity

$$V_c = [GM/(r-2m)]^{1/2} \text{ where } r > 3m \quad (1)$$

Binding energy of a mass m^* in circular orbit

$$m^*\varepsilon = m^*c^2\{1-(r-2m)\,[r(r-3m)]^{-1/2}\} \quad (2)$$

Angular momentum of circular orbit per unit mass

$$h = [mc^2\, r^2/(r-3m)]^{1/2} \quad (3)$$

The maximum binding energy in circular orbit is

$$m^*c^2\,(1-2\sqrt{2}/3) = 0 \cdot 057\ m^*c^2 \sim m^*c^2/18 \quad (4)$$

which occurs at $r = 6m$ and $h = \sqrt{12}\ mc$ (equation (5)). This orbit is also the circular orbit of least angular momentum. The period of the circular orbit as seen from infinity is $(2\pi r/V_c)\,(1-[2m/r])^{-1/2} = 2\pi\,(r^3/GM)^{1/2}$ (equation (6)). The maximum wavelength change toward the blue visible from infinity is $\lambda_0/\lambda = 2^{1/2}$ for a stable circular orbit while for a stable parabolic orbit it is $\lambda_0/\lambda = 1 + 2^{-1/2}$. For parabolic orbits that will disappear down the Schwarzschild throat the greatest blueward change is seen from $r = 27m/8$ when $\lambda_0/\lambda = 1 \cdot 77$. There is no possibility of synchrotron-like blue-shifts.

Numerical values are: $V_c = 200\ (M_7/r)^{1/2}$ km s^{-1} at r pc. $12m$ diameter of Schwarzschild mouth $= 1 \cdot 6 \times 10^{13}\ M_7$ cm $\simeq M_7$, AU. Roche limit for a star of density ρ^* g cm^{-3} $(R) = 4 \cdot 1 \times 10^{13}\,(M_7/\rho^*)^{1/3}$ cm $= 2 \cdot 8\,(M_7/\rho^*)^{1/3}$ AU. Greatest swallowable angular momentum per unit mass $h_0 = 4mc = 0 \cdot 5\ M_7$ pc km s^{-1}. Once the initial very low angular momentum stars have been swallowed (those with $h < h_0$) the star swallowing rate rapidly declines to a negligible trickle of about 10^{-8} stars yr^{-1}. Because the Roche limit is near the Schwarzschild mouth we may likewise neglect the tearing apart of stars by tides. Spectroscopic velocity dispersion of the same energy as the circular orbit $\sigma^2 = \frac{1}{3}\,V_c^2$, that is, $\sigma = 116\ (M_7/r)^{1/2}$ km s^{-1}. Period of circular orbit at r pc $= 3 \times 10^4\ (r^3/M_7)^{1/2}$ yr.

It is thus by no means impossible that there are collapsed masses in galactic nuclei. Considerable support for the notion comes from a detailed consideration of what happens when a cloud of gas collects in the galactic nucleus. (This was first considered by Salpeter, who also derived the 0·057 Fc^2 power output[5] which is now described.)

Gas Swallowing

The total mass loss from all stars in a galaxy will be roughly 1 $M_\odot$ per year. A fraction of this accumulates in galactic nuclei, which are the centres of gravitational attraction. There is dissipation when gas clouds collide, due to shock waves that radiate the energy of collision. For a given angular momentum the orbit of least energy is circular, so we must expect gas to form a flat disk held out from the centre by circular motion. Such a differentially rotating system will evolve due to "friction" just as described earlier for a dying quasar. Nothing happens in the absence of friction so the energy is liberated via the friction. In the cosmic situation molecular viscosity is negligible and it is most probable that magnetic transport of angular momentum dominates over turbulent transport just as it does in Alfvén's theory of the primaeval solar nebula. To give a sensible model of the swallowing rate we must estimate the magnetic friction and investigate what happens to the energy acquired by the magnetic field. Before doing this let us assume a conservative swallowing rate of 10^{-3} $M_\odot$ per year and work out the power available through magnetic friction. We assume that this mass flux is processed down through the circular orbits until it reaches the unstable orbit at $r=6m$. We shall assume that the flux is swallowed by the throat without further energy loss. The power for a mass flux F is then $0{\cdot}057\ Fc^2 = 3{\cdot}5\times 10^{42}$ erg s$^{-1}\simeq 10^9$ $L_\odot$ where the values are for $F=10^{-3}$ $M_\odot$ yr^{-1} and $L_\odot$ is the power of the energy output of the Sun. This sort of power emitted as light could just noticeably brighten a nucleus. If a fraction were emitted in the radio region the nucleus would be a radio source. Clearly it only requires a mass flux of 1 $M_\odot$ yr^{-1} and a conversion into light at 10 per cent efficiency for the nucleus to equal the stellar light output from the whole galaxy. Can this be the explanation of the Seyfert galaxies ?

We now return to the model for the magnetic transfer of angular momentum in a disk. As a magnetic field B is sheared by an initially perpendicular displacement. The component across the shear is left unchanged but the component down the shear is progressively amplified. We may therefore expect the magnetic field in a shearing medium to be progressively amplified until it somehow changes either the motion or its own configuration. The magnetic field first has a significant effect when it can bow upwards and downwards out of the differentially rotating disk leaving the material to flow down the field lines and so collect into clouds in the disk[6]. For significant cloudiness to result, the magnetic field pressure $B^2/8\pi$ must equal the turbulent and gas pressures ρc_s^2. Here ρ is the density and c_s^2 the combined velocity dispersion of microscopic and molecular motion. The thickness of the disk $2b$ is determined by the balance between the gravity of the central mass and the non-magnetic pressure, $GMr^{-3}z\rho = -c_s^2\,\partial\rho/\partial z$, which gives

$$\rho = \rho_0 \exp\left[-z^2/2b^2\right] \text{ where } b^2 = r^3c_s^2/GM \qquad (7)$$

The local shear rate is given by Oort's constant A

$$A = -\tfrac{1}{2}\, r\, \mathrm{d}(v/r)/\mathrm{d}r = \tfrac{3}{4}\,(GM/r^3)^{1/2} \qquad (8)$$

$$\text{Hence } \frac{B^2}{8\pi} \simeq \rho_0 c_s^2 = \frac{16}{9}A^2b^2\rho_0 = \left(\frac{16}{9\sqrt{2\pi}}\right)A^2b\Sigma \qquad (9)$$

Here Σ is the surface density of matter in the disk, $\Sigma=\sqrt{2\pi}\, b\ \rho_0$.

On multiplication by b^3, equation (9) tells us that the magnetic field energy in a volume b^3 is about equal to the kinetic energy of the shearing in the same region. Equation (9) may be obtained approximately from a dimensional analysis of a general shearing sheet of highly conducting material. As such, it claims general validity, so it is interesting to apply it to the gas in the neighbourhood of the Sun. The very reasonable result, $B=4\times 10^{-6}$ G, restores confidence in this rather inadequate treatment.

We are now in a position to estimate the magnetic frictional force per unit length in a shearing disk. The Maxwell stress $B_x . B_y/(4\pi)$ acts over a thickness of about $2b$. Once the medium has broken up into clouds the magnetic field will be rather chaotic but the shearing will still give it some systematic tendency to oppose the shear. The greatest possible value of B_xB_y for given $[\mathbf{B}]$ is $B^2/2$, so we estimate $B^2/4$ as a typical value in a sense directed to oppose the shear. The force per unit length is then

$$bB^2/8\pi = [16/(9\sqrt{2\pi})]\,A^2b^2\Sigma \qquad (10)$$

Unlike a normal viscous drag, the force here depends on the square of A, the rate of shear. This is reasonable because the shear itself is needed to build up the magnetic field which eventually opposes it. We shall use this estimate of the friction to make a model of the disk, but it is important to consider first where the energy goes. We must consider why the field is not further amplified by further shearing once it has reached the value estimated. Once the medium has split into clouds each cloud acts like a magnet. As the medium is sheared these magnets try to re-align themselves to take up a configuration of minimum energy. The intercloud medium, although dominated by the magnetic field, is still a highly conducting medium, however. In free space re-alignment of the magnets involves reconnexion of magnetic field lines through neutral points. This cannot happen in a force-free, strictly perfect, conductor. Rather one may show that neutral sheets develop with large sheet currents flowing through them. This situation does not really happen; any small resistivity causes a return to the reconnecting case and it is of the greatest interest to see how a plasma of particles behaves near a neutral point at which reconnexion takes place. (In the frame in which the neutral point is fixed this involves an electric field E. The mechanism of acceleration is basically that of Syrovatskii[7].) Tritton at this observatory has been studying with me the details of an exact model. To reconnect a finite flux in a finite time through a point at which $B=0$, it is clear that the lines of force must move infinitely fast at the neutral point. They are not material lines and there is nothing wrong with this; even the normal formula for their velocity $v=c\,(\mathbf{E}\times\mathbf{B}/B^2)$ gives superluminous velocities when $|\mathbf{E}\times\mathbf{B}|>B^2$ (that is, for $|E|>|B|$ in gaussian units assuming they are perpendicular). The particles gyrate about their field lines until the lines move with the velocity of light; thereafter the electric field predominates, the magnetic field is too weak to change anything significantly and the particles are electrostatically accelerated. The potential drop follows directly from Faraday's law

$$\Phi = -\frac{1}{c}\,\mathrm{d}N/\mathrm{d}t$$

where $\mathrm{d}N/\mathrm{d}t$ is the rate of flux reconnexion. To calculate this e.m.f. we estimate the reconnexion rate as a flux of $2b^2B$ in a time of $(2A)^{-1}$. This gives an e.m.f. of $4b^2AB/c$ (equation (11)). We shall see that these e.m.f.s are of the order of 10^{12} V. Because I believe that this can be the primary source of dissipation for the highly conducting disk I deduce that the energy of dissipation can be converted directly into cosmic rays in the GeV range. We have now good reason to believe that galactic nuclei ought to be radio sources because such cosmic rays will clearly radiate by the synchrotron mechanism in the magnetic field. In summary, we expect from a shear $2A$ the generation of a magnetic field given by equation (2), a shearing force given by equation (3), and a power p per unit area dissipated into cosmic rays of energy up to $4b^2AB/c$ given by $p=2AbB^2/8\pi=(32/(9\sqrt{2\pi}))\ A^3b^2\Sigma$ (equation (12)). These formulae have very general application in the astrophysics of the Galaxy, in the early history of the solar nebula and in the origin of peculiar A stars from binaries. There is, however, one caveat: the reconnexion energy only goes into cosmic rays if the density is low enough in the reconnecting region. If the particle being accelerated has a collision before it reaches the r.m.s. velocity, then it cannot run away to high energy; instead the reconnexion energy is dissipated by ohmic heating.

If m_A, m_p are the masses of the accelerated particle and the proton, then the condition for acceleration is

$$e\, m_A^{-1} E\, t_s > \left(\frac{3}{2}\, kT m_p^{-1}\right)^{1/2}$$

where t_s is the time between scatterings of the particles A. This time may be taken from Spitzer's book[8] to be approximately

$$t_s = m_A\, \rho^{-1}\, T^{3/2}\, m_A/(m_A + m_p) \text{ seconds}$$

T is the temperature and ρ is the density at the acceleration region. Putting in our expressions for the electric field $\Phi/2b$ we obtain for our disk in c.g.s. units the acceleration condition

$$\rho < 10^{-24}\left(\frac{2m_A}{m_A + m_p}\right) bABT$$

Notice the density for proton acceleration can be 918 times that for electron acceleration.

The Steady Model Disk

We look for a steady state of gas swallowing with a mass flux F. We shall assume that a small fraction of the rotational energy is converted into random motions so that $c_s \propto V_c$. Sensible values would be $c_s = 10$ km s^{-1} when $V_c = 200$ km s^{-1} so we write $c_s = (1/20)\, x\, V_c$, where x is of order unity but might be a weak function of r. The couple on the material inside r due to magnetic friction is from equations (10), (8) and (7)

$$g = 2\pi r^2 b B^2/8\pi = \sqrt{2}\pi\, c_s^2\, r^2\, \Sigma \qquad (13)$$

Apart from the trickle of angular momentum $\sqrt{12}\, mcF$ into the singularity g must be balanced in a steady state by the inward flux of angular momentum carried by the material. Thus, using equations (3) and (5)

$$g = \sqrt{12}\, mcF + F\,[mc^2 r^2 (r - 3m)^{-1}]^{1/2} \text{ for } r > 6m \qquad (14)$$

that is

$$g \simeq F\,(GMr)^{1/2} \qquad (15)$$

Because $g \propto r^{1/2}$ we see from equation (13) that $\Sigma \propto r^{-3/2}$ $c_s^{-2} \propto r^{-3/2}$ $V_c^{-2} \propto r^{-1/2}$ and that the radial velocity

$$V_r = F\,(2\pi r \Sigma)^{-1} \qquad (16)$$

The power $2\pi r p(r) \delta r$ liberated into heat or cosmic rays in the region between r and $r + \delta r$ is $-F\mathrm{d}\varepsilon/\mathrm{d}r$ where $-\mathrm{d}\varepsilon/\mathrm{d}r$ is the derivative of the binding energy in circular orbit given by equation (2).

$$p(r) = \frac{Fc^2}{4\pi r}\,\frac{m\,(r - 6m)}{[r(r - 3m)]^{3/2}} \quad \text{for } r \geq 6m \qquad (17)$$

$$p(r) \simeq FGM/(4\pi r^3) \qquad \text{for } r \gg m \qquad (18)$$

For a given total mass and radius of the disk, a given Schwarzschild mass and choice of the parameter $x \simeq 1$, we have now a unique model of the disk including the cosmic ray power, the heating per unit area, the magnetic field and the mass flux. Rather than give the total mass and radius of the disk, we shall determine everything in terms of the mass flux. Any chosen maximum radius then determines the total mass within. We shall calculate two temperatures T_1 and T_2 as follows. T_1 is the temperature that the disk would have if it radiated as a black body just the power per unit area that is locally generated. Thus $T_1(r) = (P(r)/(2\sigma))^{1/4}$ where σ is Stefan's constant and the factor 2 arises because the disk has two sides. $T_2(r)$ is the ambient black body temperature at a distance r from a source the total power of which is the total power of one disk. Thus

$$T_2(r) = [P/(16\pi\sigma r^2)]^{1/4}$$

We now give the numerical values in terms of the following variables: F_{-3} the mass flux in units of $10^{-3}\, M_\odot$ per year, M_7 the Schwarzschild mass in units of $10^7\, M_\odot$, x the ratio of the turbulent velocity to 1/20 V_c. r_0, m_0 the running variable r in units of 1 pc and GM/c^2 respectively. From equation (15) couple $g = 4 \times 10^{48}\, r_0^{1/2} F_{-3} M_7^{1/2}$ $= 2{\cdot}6 \times 10^{45}\; m_0^{1/2}\; F_{-3}M_7$ g cm^2 s^{-1} $(m_0 \gg 6)$. From equation (13) surface density $\Sigma = 0{\cdot}16\, r_0^{-1/2}\, x^{-2}\, F_{-3}M_7^{-1/2}$ $= 240\; m_0^{-1/2}\; x^{-2}\; F_{-3}M_7^{-1}$ g cm^{-2} $(m_0 \gg 6)$. $\int_0^r 2\pi r \Sigma \mathrm{d}r =$ $6{\cdot}7 \times 10^{36}\; r_0^{3/2}\; x^{-2}\; F_{-3}M_7^{-1/2}$ g $= 3{\cdot}4 \times 10^3\; r_0^{3/2}$ solar masses. From equation (7) $b/r = 0{\cdot}05x$. $\rho_0 = (2\pi)^{-1/2}\Sigma/b =$ $4{\cdot}6 \times 10^{-19}\, r_0^{-3/2}\, x^{-3}\, F_{-3}M_7^{-1/2} = 1{\cdot}4 \times 10^{-9}\, m_0^{-3/2}\, x^{-3}\, F_{-3}M_7^{-2}$ g cm^{-3} $(m_0 \gg 6)$. From equation (13), $B = 3 \times 10^{-3}\; r_0^{-5/4}$ $x^{-1/2}\; F_{-3}^{1/2} M_7^{1/4} = 2{\cdot}7 \times 10^5\; m_0^{-5/4}\; x^{-1/2}\; F_{-3}^{1/2} M_7^{-1}$ G $(m_0 \gg 6)$. From equation (18), $p(r) \simeq 0{\cdot}22\, r_0^{-3}\, F_{-3}M_7$ ergs cm^2 s^{-1} $(r \gg 6m)$. Notice that the power is strongly concentrated towards the centre, where we need the accurate relativistic formula (17)

$$p(r) = 0{\cdot}22\; r_0^{-3}\left[\frac{1 - 2{\cdot}6 \times 10^{-6}\, M_7\, r_0^{-1}}{(1 - 1{\cdot}3 \times 10^{-6}\, M_7\, r_0^{-1})^{3/2}}\right] F_{-3}\, M_7$$

$$= 2{\cdot}6 \times 10^{18}\left[\frac{1 - 6\, m_0^{-1}}{(1 - 3\; m_0^{-1})^{3/2}}\right] m_0^{-3}\; F_{-3}\; M_7^{-2} \text{ erg cm}^{-2}\text{ s}^{-1}$$

Total power $P = 0{\cdot}057\; Fc^2 = 3{\cdot}2 \times 10^{42}\; F_{-3}$ erg s$^{-1} \sim 10^9\, L_\odot$

$$T_1(r) = 6{\cdot}7\, r_0^{-3/4}\left[\frac{1 - 2{\cdot}6 \times 10^{-6}\, M_7\, r_0^{-1}}{(1 - 1{\cdot}3 \times 10^{-6}\, M_7\, r_0^{-1})^{3/2}}\right]^{1/4} F_{-3}^{1/4}\; M_7^{1/4}$$

$$= 3{\cdot}7 \times 10^5\, m_0^{-3/4}\left[\frac{1 - 6\, m_0^{-1}}{(1 - 3\; m_0^{-1})^{3/2}}\right]^{1/4} F_{-3}^{1/4}\; M_7^{-1/2} \text{ K}$$

$$(m_0 \geq 6)$$

$$T_2(r) = 100\, r_0^{-1/2}\, F_{-3}^{1/4} = 1{\cdot}6 \times 10^5\, m_0^{-1/2}\, F_{-3}^{1/4}\, M_7^{-1/2} \text{ K}$$

Maximum cosmic ray energy $e\Phi = 1{\cdot}5 \times 10^{13}\; r_0^{-3/4}\; x^{3/2}$ $F_{-3}^{1/2}\; M_7^{3/4} = 10^{18}\; m_0^{-3/4}\; x^{3/2}\; F_{-3}^{1/2}$ eV. Period of circular orbit (seen from infinity) $3 \times 10^4\; r_0^{3/2}\; M_7^{-1/2}$ yr $= 9{\cdot}8 \times 10^{-6}$ $m_0^{3/2}\; M_7$ yr. Circular velocity $V_c = 200\; r_0^{-1/2}\; (1 - 9{\cdot}7 \times 10^{-7}$ $M_7\; r_0^{-1})^{-1/2} = 3 \times 10^5\; m_0^{-1/2}\; (1 - 2\; m_0^{-1})^{-1/2}$ km s^{-1} $(m_0 > 6)$. Radial velocity $V_r = 0{\cdot}2\; r_0^{-1/2}\; x^2\; M_7^{1/2} = 3 \times 10^2\; m_0^{-1/2}\; x^2$ km s^{-1} $(m_0 \gg 6)$. Condition for electron acceleration is $\rho < 3 \times 10^{-20}\, T_4\, r_0^{-7/4} x^{1/2} F_{-3}^{1/2} M_7^{3/4}$ and for proton acceleration $\rho < 3 \times 10^{-17}\; T_4$, etc., where T_4 is the temperature in the acceleration region in units of 10^4 K and ρ is the density. Acceleration will be between the clouds so at any region of the disk we should probably take $\rho_0/10$ for ρ (and $T_4 \sim 1$ unless the ambient temperature T_1 is greater.)

Diameter of Schwarzschild mouth $12m = 5{\cdot}7 \times 10^{-6}\; M_7$ pc, that is, $m_0 = 12$. Diameter of region producing half the power $44m = 2{\cdot}1 \times 10^{-5}\; M_7$ pc, that is, $m_0 = 44$. Rotational period at that radius $9 \times 10^{-4}\; M_7$ yr $= 0{\cdot}32\; M_7$ days. Inward movement time r/V_r at that radius $1{\cdot}6 \times 10^{-1}\; M_7$ x^{-2} yr $\sim 59\; M_7$ days.

It should by now be clear that with different values of the parameters M_7 and F_{-3} these disks are capable of providing an explanation for a large fraction of the incredible phenomena of high energy astrophysics, including galactic nuclei, Seyfert galaxies, quasars and cosmic rays. The next section is therefore devoted to predicting the spectra.

Spectrum

The maximum temperature is at $m_0 = 7{\cdot}05$ and is $T_1 =$ $6{\cdot}6 \times 10^4\; F_{-3}^{1/4}\; M_7^{-1/2}$ K. The medium will be optically thick for $\Sigma = 90\; x^{-2}\; F_{-3}\; M_7^{-1}$ g cm^{-2}. The disk is in danger of becoming optically thin around $\Sigma = 1$, but there the temperature has fallen to $T_2 \sim 700$ K so dust will take over as a source of opacity (this may not happen for large M_7). Our standard model with the parameters all at unity will provide opacity out to about a parsec or so. Because all but the centre of our disk obeys a law $T = Ar^{-2a}$ with $a = 4$ in the outer parts where T_2 is relevant and $a = 8/3$ in the inner parts where T_1 is relevant, we study the radiation from disks with such power law tem-

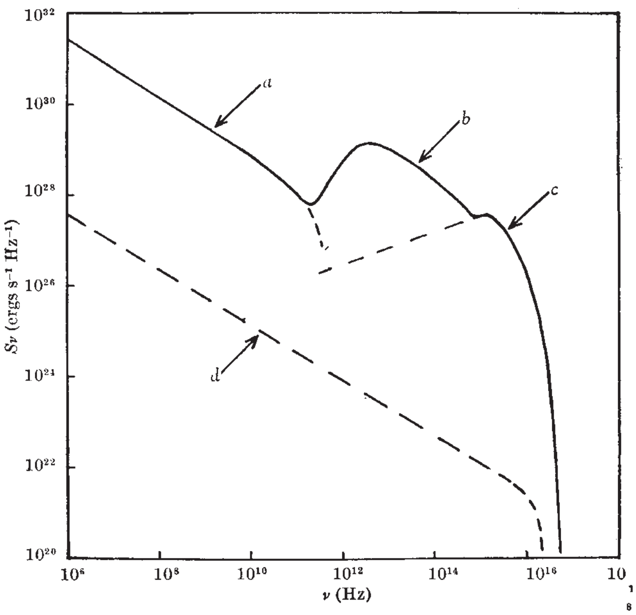

Fig. 1. The emitted spectrum of disk and synchrotron radiation for the standard model. The flux from Sagittarius *A* is weaker by a factor 100, indicating only 1 per cent efficiency of the proton synchrotron. *a*, Proton synchrotron; *b*, outer disk; *c*, central disk; *d*, electron synchrotron.

perature distributions. The total emission at frequency ν is given by

$$S_\nu = \int_0^\infty \frac{c}{4} u_\nu\,(T(r))\; 4\pi r \mathrm{d}r \;=\; \frac{8\pi^2 h}{c^2}\int_0^\infty \frac{\nu^3\, r\mathrm{d}r}{\exp(h\nu/kT)-1}$$

Writing $x = h\nu/kT = h\nu r^{2a}(kA)$ we find

$$S_\nu = \frac{4\pi^2 h}{c^2}\left(\frac{kA}{h}\right)^{a}\int_0^\infty \frac{a\,x^{a-1}}{\mathrm{e}^x-1}\,\mathrm{d}x\;\nu^{3-a}$$

where $$\int_0^\infty \frac{a\,x^{a-1}}{\mathrm{e}^x-1}\,\mathrm{d}x = a\,\Gamma(a)\zeta(a)$$

Thus for $a = 8/3$ we have $S_\nu \propto \nu^{1/3}$ while for $a = 4$ we have $S_\nu \propto \nu^{-1}$. Before trying to use these formulae it is important to find out at what radius the main contributions to S_ν arise. For $a < 1$ the main contributions come from radii close to those for which $h\nu \sim kT$. We may therefore deduce that for our standard model $S_\nu \propto \nu^{-1}$ when 100 K $< h\nu/k <$ 3,000 K and $S_\nu \propto \nu^{1/3}$ when 3×10^4 K $< h\nu/k < 10^5$ K, and that for frequencies corresponding to temperatures of 10^5 K or greater the system shines like a black body of 10^5 K. In practice it is known that at least for Seyfert galaxies the reddening by dust takes a large fraction of the energy out of the ultraviolet and replaces it in the infrared. Because I have no theory for the amount of dust at each radius I cannot predict the final optical spectrum in detail. But because dust evaporates at a thousand degrees or so it would seem likely that the radiation should be peaked on the red side of the corresponding frequency. Fig. 1 shows the details of the emitted "black body" radiation. It is clear that fluorescence from the ultraviolet will mean that the optical spectrum should be full of emission lines. Those arising from where the disk has ambient temperatures near 2×10^3 K come from regions $r_0 \simeq 10^{-3}$ where the circular velocities are $V_c \simeq 6\times10^3$ km s^{-1}. These emission lines should therefore be very broad. We may expect that the real disk is not steady, although exact periodicity due to a source in orbit is unlikely. Rather, we should expect variations on a time scale given by $2r/V_r$ at $m = 22$ (120 days) because that is the time scale in which the material flux can vary over the region in which most of the flux is emitted. Using our theory in the most straightforward way it is clear that more power is used in accelerating protons than is used in accelerating electrons. Protons can be accelerated in a density 918 times as great as that in which the electron acceleration can operate. Density in our model behaves like $r^{-3/2}$ while total power between $3/2\,r$ and $1/2\,r$ behaves like r^{-1}. We deduce that the proton power is about 10^2 times the electron power. The steady state spectrum is easily determined. From our accelerator we expect energy proportional to potential drop. Because particles start from all points the energy spectrum ejected by the accelerator is uniform up to the maximum energy, $\mathrm{e}\Phi \sim 10^{13}$ eV.

Each particle of energy E radiates at a rate proportional to E^2. If there were constant monoenergetic injection into the medium, the flux of particles downwards in energy would be constant. Thus the number per unit area at any E less than the injection energy would follow the law $N(E) = KE^{-2}$ for $E < E_{\max} \equiv E_{\mathrm{m}}$. This law is only slightly modified by our uniform injection at all energies up to $E_{\max}$. It is

$$N(E) = K\,(1 - E/E_{\mathrm{m}})\,E^{-2} \quad E < E_{\mathrm{m}}$$

where K is related to the total power of injection power per unit area p by

$$K = \frac{2p}{E_{\mathrm{m}}}\;\frac{9\,m_{\mathrm{A}}^2\,c^7}{4e^2}\,B^{-2}$$

Because the power law is near E^{-2} except close to $E_{\max}$, we expect the γ of synchrotron radiation theory to be close to two, and the corresponding spectrum to be close to $S_\nu \propto \nu^{-\alpha}$ with $\alpha = \frac{1}{2}$. It is possible to work out a better approximation using the δ function approximation to the frequency spectrum of a single electron[9]. For our disk model the flux is

$$S_\nu = \int_{r_{\mathrm{A}}}^{\infty}\int_0^{E_{\mathrm{m}}} \frac{2p}{E_{\mathrm{m}}}\;\delta\,(\nu - \nu_{\mathrm{m}})\left(1 - \frac{E}{E_{\mathrm{m}}}\right)\mathrm{d}E\;2\pi r\,\mathrm{d}r$$

where r_{A} is the least radius at which the particles can be accelerated and

$$\nu_{\mathrm{m}} = 0{\cdot}07\left(\frac{2}{3}\right)^{1/2}\;\frac{e}{m_{\mathrm{A}}^3\,c^5}\;BE^2$$

Using our power laws for $p(r)$, $E_{\mathrm{m}}(r) \equiv \mathrm{e}\Phi$, $B(r)$, we obtain

$$S_\nu = 3{\cdot}8\times10^{28}\left(\frac{m_{\mathrm{A}}}{m_{\mathrm{p}}}\right)^{12/11}\nu_9^{-7/11}$$

$$\left[1 - \frac{1}{11}\left(14 - 3\left(\frac{\nu_9}{\nu_{\mathrm{A}}}\right)^{1/2}\right)\left(\frac{\nu_9}{\nu_{\mathrm{A}}}\right)^{3/22}\right] x^{-10/11}\,F_{-3}^{\;5/11}\,M_7^{\;4/11}$$

where this formula holds for $\nu < \nu_{\mathrm{A}} \equiv \nu_{\mathrm{m}}(r_{\mathrm{A}})$ and ν_9 is ν in units of GHz. This formula has assumed that all the power dissipated goes into fast protons or electrons as the case may be. In regions where both electrons and protons are accelerated the power should obviously be divided by two. In practice it is probable that S_ν should be reduced by some efficiency factor because probably only a fraction of the total reconnexion energy really gets into fast particles. The radius of the source at frequency ν is about

$$r_0 = 0{\cdot}7\;\nu_9^{-4/11}\left(\frac{m_{\mathrm{A}}}{m_{\mathrm{p}}}\right)^{-12/11} x^{10/11}\,F_{-3}^{\;6/11}\,M_7^{\;7/11},\;\nu < \nu_{\mathrm{A}}$$

Notice that the fast electrons can only be produced much further out than the protons but that they nevertheless produce radiation to much higher frequencies.

Comparison with Observations

In the Galaxy it is not clear that the circular velocity near the centre falls below 200 km s^{-1}, but the OH observations do suggest velocities as low as 100 km s^{-1} within 70 pc of the centre. This indicates a nuclear mass of $M_7 \sim 3$ for the central singularity. The size and flux from Sagittarius *A* are in rough accord with our estimate of the synchrotron spectrum. An infrared flux found at

100 μm could be due to dust from an ultraviolet source radiating 10^9 $L_\odot$, so the flux of mass into the throat must be around $F_{-3}=1$. The general level of activity observed at radio wavelengths close to the nucleus indicates that high energy phenomena are involved[10].

The Magellanic clouds have no nucleus. *M*31 has a strong radio source rather larger but weaker than that found in the Galaxy[11]. Code has discovered strong ultraviolet emission from the nucleus, which Kinman finds to have a large mass-to-light ratio and to contain some 10^8 solar masses. This suggests a small mass flux into the centre of *M*31 and only a very small ultraviolet disk about the Schwarzschild mouth. *M*32 has a nucleus which is not a radio source but the system is very deficient in gas. We suggest that this system has a Schwarzschild mass but the Galaxy has run out of gas and left it hungry. *M*82 has had a recent violent radio explosion and an infrared nucleus with a small bright radio source in the centre. I suspect $M_7 \sim 3$ and $F_{-3} \sim 10$ but that F_{-3} was larger in the recent past. *M*81 has a very small flux but an intense radio source at its nucleus. I suggest that it is intermediate between the Galaxy and *M*31. *M*87 is the nearest really bright radio galaxy. Luckily the velocity dispersion in its nucleus has been measured[12]. We can therefore measure M_7 with some pretence of accuracy to be about 4×10^{10} $M_\odot$, that is, $M_7 = 4 \times 10^3$. Over 10^{10} yr it would take an F_{-3} of 4×10^3 to build such an object. This is probably the nearest old dead radio-bright quasar. It is only a shadow of its former self, as F_{-3} has declined severely as the gas has run out. Its electron synchrotron still produces copious X-rays[12], however. *NGC* 4151 is a Seyfert galaxy, and M_7 need not be greater than 3, or more likely 30, but F_{-3} is high because there is still much gas in the central regions. Seyfert galaxies that are active have $F_{-3} = 10^3$ but probably are only active at this flux level for one-hundredth of the time. The breadths of the wings of the Balmer lines are 6,000 km s^{-1}—I suggest that these are Doppler widths[14]. *NGC* 4151 is a strong infrared source.

Quasars

When F_{-3} achieves large values $\sim 10^6$ or 10^7 (that is, $10^{3 \cdot 4}$ $M_\odot$ yr^{-1}) the mass of the Schwarzschild throat rapidly builds up to 10^9–10^{10} $M_\odot$. When galaxies first formed there was this amount of gaseous material in them. Large proto-galaxies rapidly achieved large Schwarzschild throats and greedily swallowed gas. It is clear that the right energy is available and by making M close to 10^{10} $M_\odot$ we lower the densities close to the Schwarzschild throat. This allows the radio phenomena to occur closer to the singularity where more of the power is.

Note added in proof. Low's recent observations of the galactic centre at 100 μm (reported at the Cambridge conference on infrared astronomy, 1969) suggest that F_{-3} for the galaxy is nearer 10^{-2} than 1. A dust model by Rees can explain the infrared observations assuming a single central source of visible or ultraviolet light. The light pressure from such sources will expel dusty material from the nuclei of Seyfert galaxies causing the observed outflow as suggested by Weymann. Such a mechanism could cut off the flux F_{-3} and therefore produce the changes in the emitted flux. The light pressure may drive the dust out of the nucleus so that no dust is ever swallowed. This could leave a great enhancement of dust in the surroundings of the nucleus corresponding to the dust content of all material swallowed in the past. A violently active outburst of such a nucleus would then be associated with the expulsion of great swathes of dust such as those seen across several radio galaxies.

The proton synchrotron radiation discussed here is probably replaced in practice by synchrotron radiation from electron secondaries and X and γ-ray bremsstrahlung corresponding to a sizable fraction of the power input into fast protons.

I thank the Astronomer Royal for discussions, and Drs Pagel, Bingham, Tritton, Rowan-Robinson, Weymann and Osterbrock for further help and encouragement.

Received July 8, 1969.

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Purification and Properties of Initiation Factor f_2

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The protein synthesis initiation factor f_2 of *E. coli* has properties in common with the chain elongation factors G and T_u and T_s.

THE initiation of protein synthesis in bacteria involves the initiator *t*RNA fMet-*t*RNA$_f$[1–5], ribosomal subunits[6–9], *m*RNA containing the initiating codon AUG[10,11], GTP[12–15], and three initiation factors[16–18]. The initiation factors are required for the formation of the chain-initiating ribosomal complex. Initiation factors f_1 and f_2 are required when the triplet AUG or synthetic polymers are used, whereas initiation factor f_3 is required in addition to f_1 and f_2 when bacteriophage RNA is used[19].

The initial binding of fMet-*t*RNA$_f$ to ribosomes takes place on the 30*S* subunit[7–9]. The formation of this complex requires initiation factors f_1 and f_2 and GTP. A 50*S* ribosomal subunit then joins the 30*S* complex to form a 70*S* complex. This junction does not require the hydrolysis of GTP[20], and it is at this step that initiation factor f_1 is released from the complex[21]. If the analogue GMPPCP is used instead of GTP, the 70*S* complex is still formed and the f_1 is still released after the junction step. The bound fMet-*t*RNA$_f$, however, is now unreactive toward puromycin or aminoacyl-*t*RNA[20,22]. GTP