

## Sigma -2

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- That's cool! (some movies)
- That's hard!! (some equations)
- Where do we start? (ICs)
- How do we go from here? (integrators)
- Analysis



Multi-wavelength view of M81



$t = 0.0036$

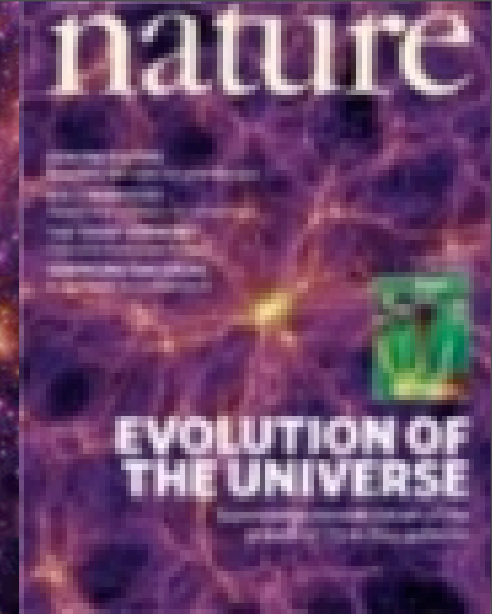


$z = 49.000$



## HYDRA: formation of structure

Springel



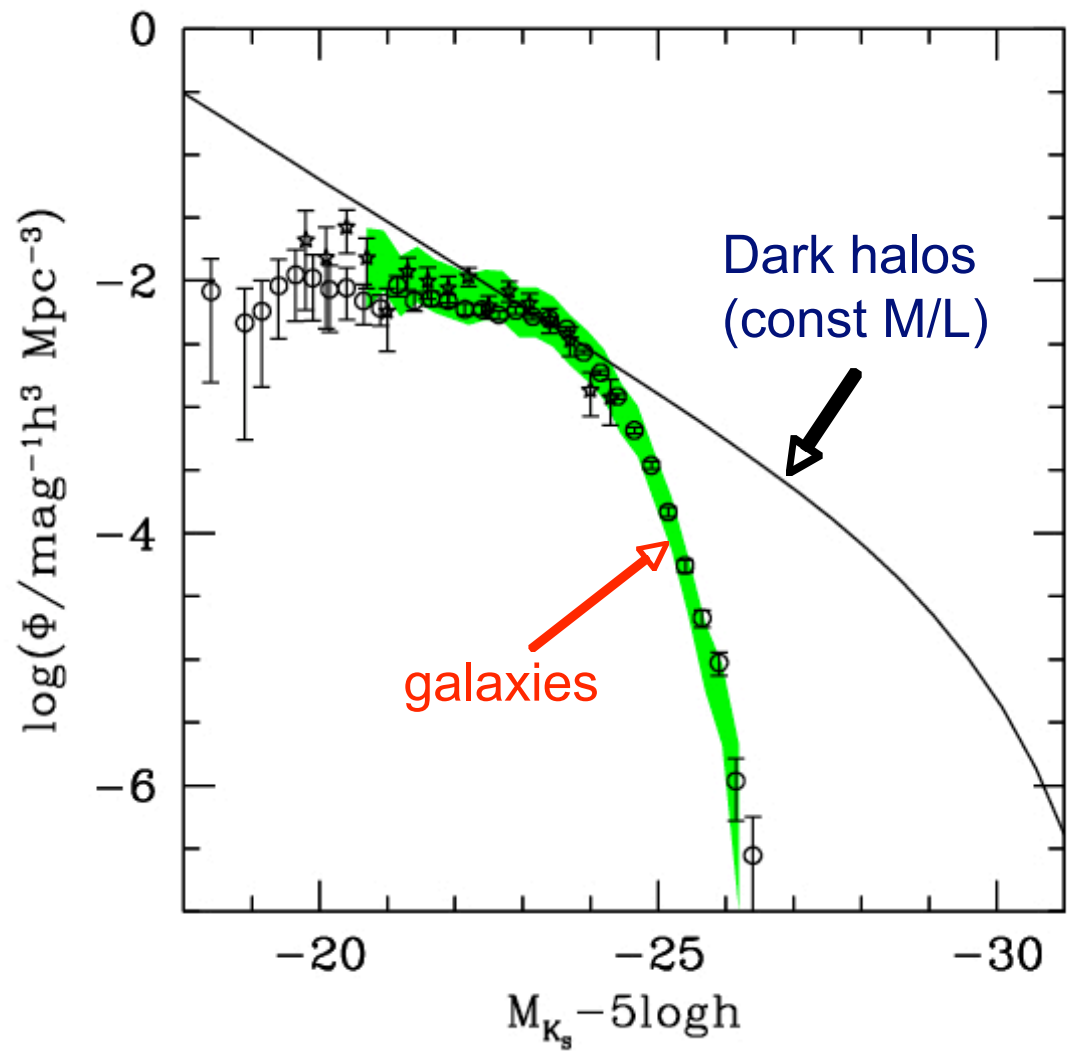
GADGET: structure of dark matter

$z = 36.0$

Springel

$M = 0.000E+00$

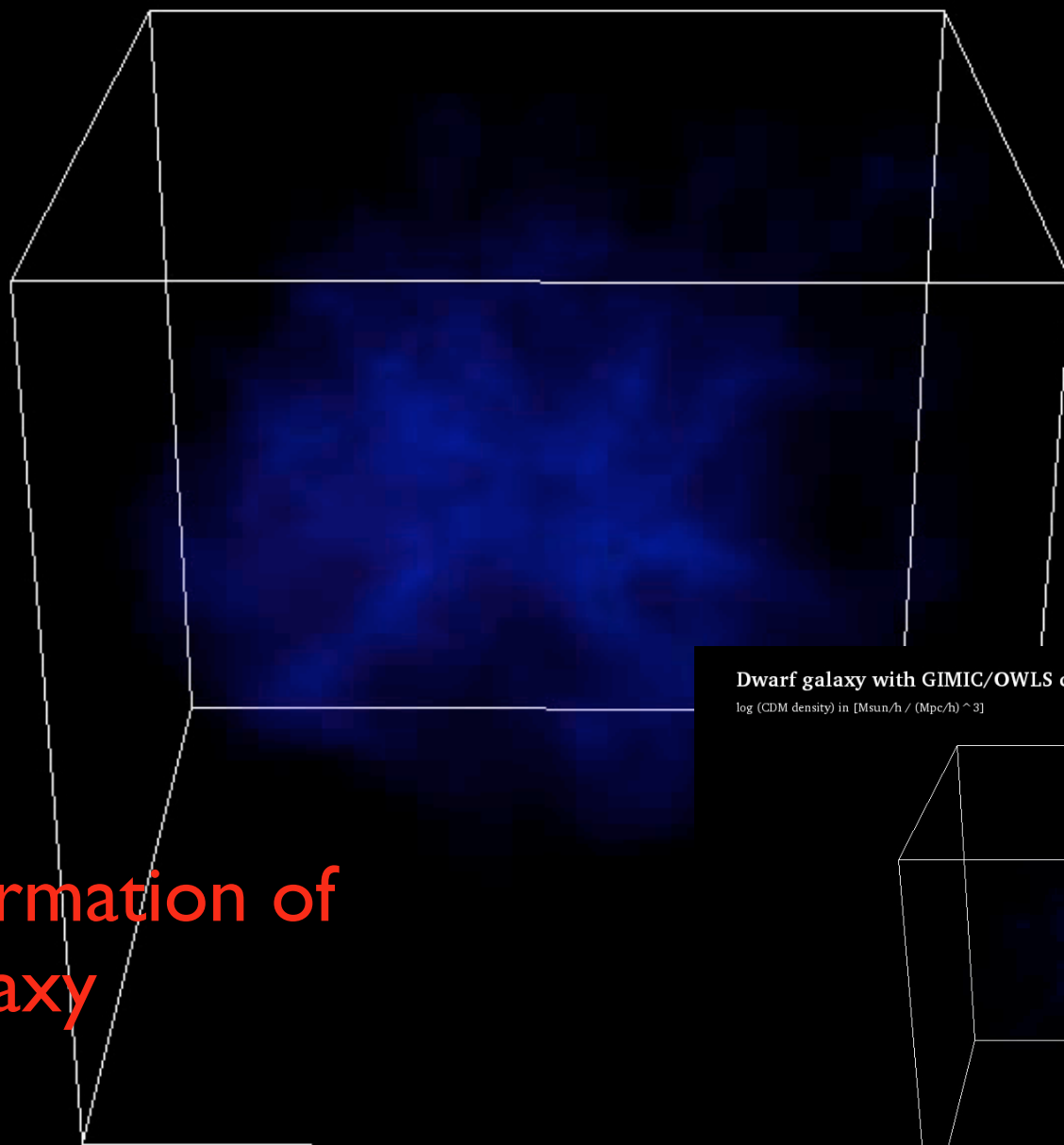
GADGET: structure of dark matter  
in the Milky Way



# Dwarf galaxy with GIMIC/OWLS code

Crain & TT

$\log(\text{Gas density})$  in  $[\text{Msun}/h / (\text{Mpc}/h)^3]$



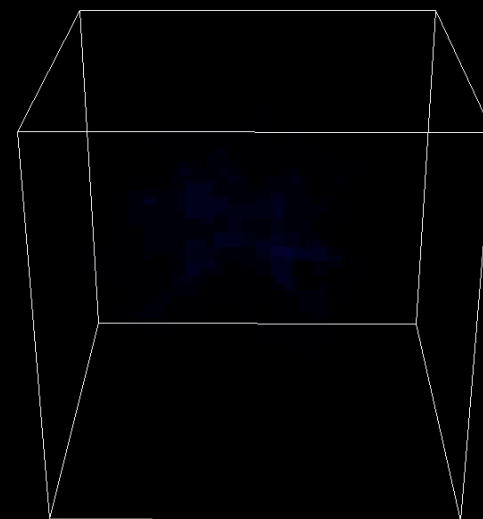
**GADGET: formation of individual galaxy**

$z = 29.888$

$L = 0.999 \text{ Mpc}/h$

## Dwarf galaxy with GIMIC/OWLS code

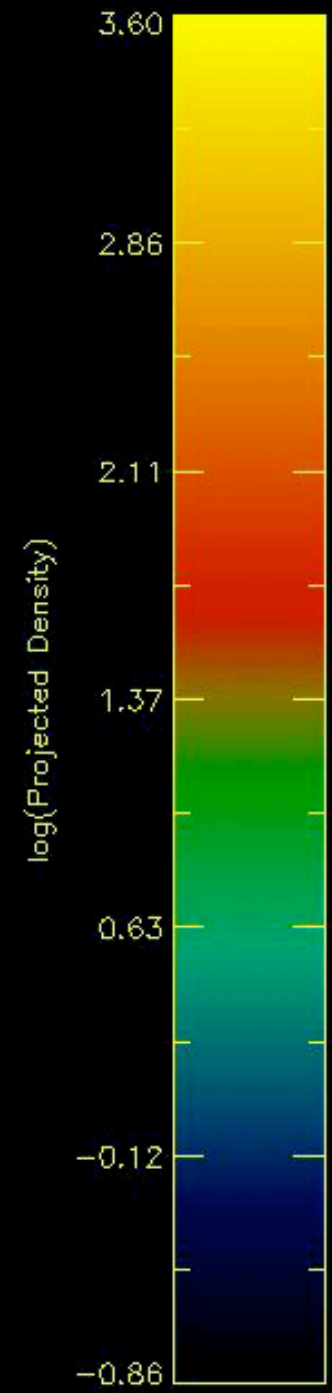
$\log(\text{CDM density})$  in  $[\text{Msun}/h / (\text{Mpc}/h)^3]$



$z = 29.888$

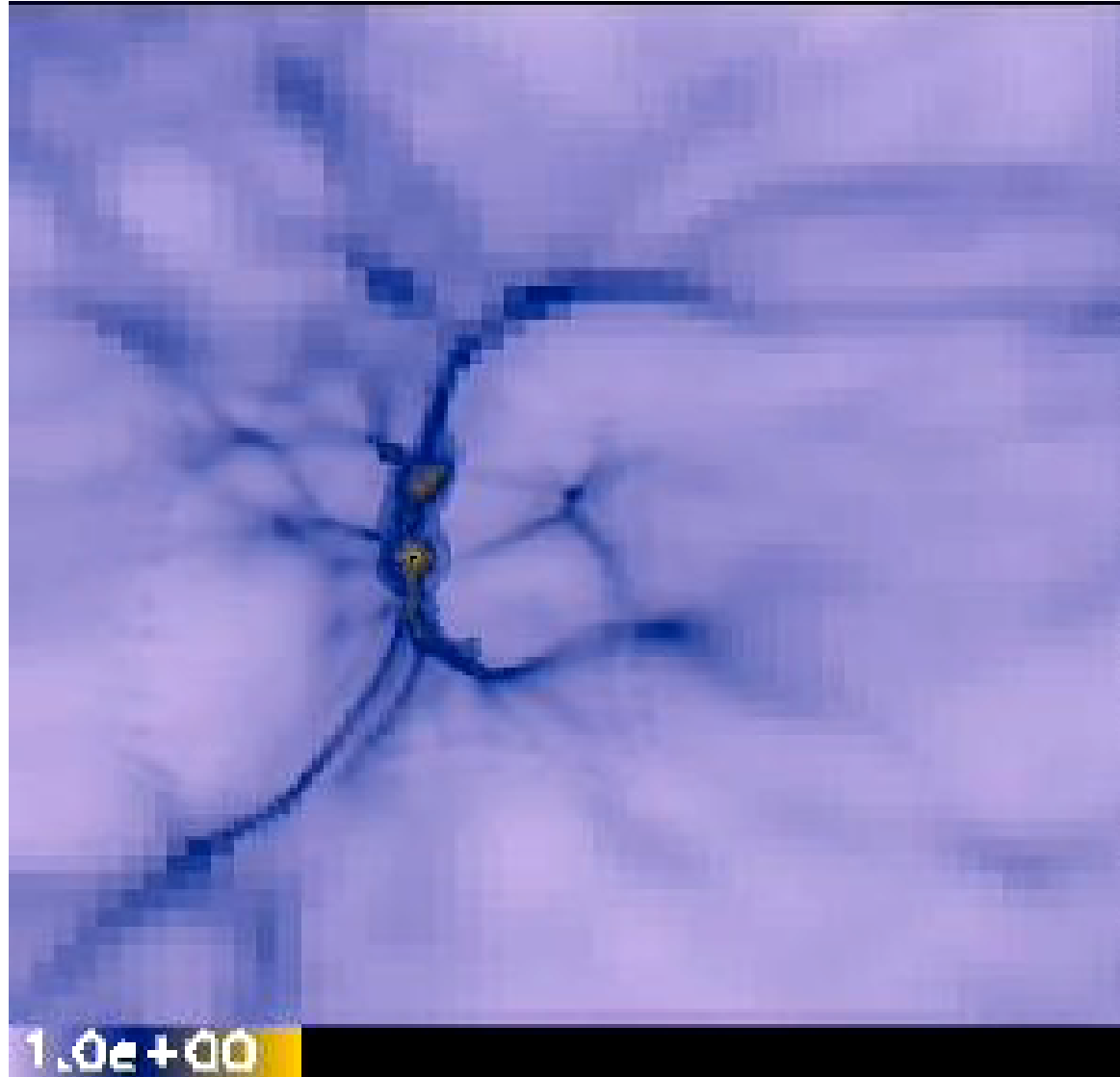
$L = 0.999 \text{ Mpc}/h$

Wise & Abel



ENZO: first galaxies

# Abel



## ENZO: first stars

Tom Theuns

# White Dwarf Deflagration

Resolution: 6 km

Initial Bubble Radius: 18 km

Ignition Offset: 42 km

Variable 1: Density [ $1.5e+07$  -  $2.0e+07$ ]

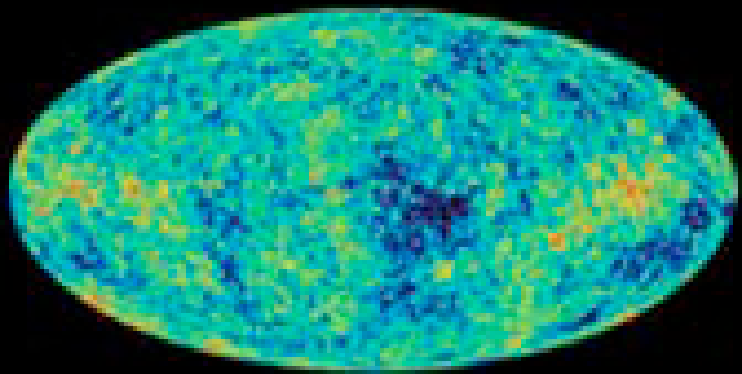
Variable 2: Reaction Progress [0.0 - 1.0]

## What we want:

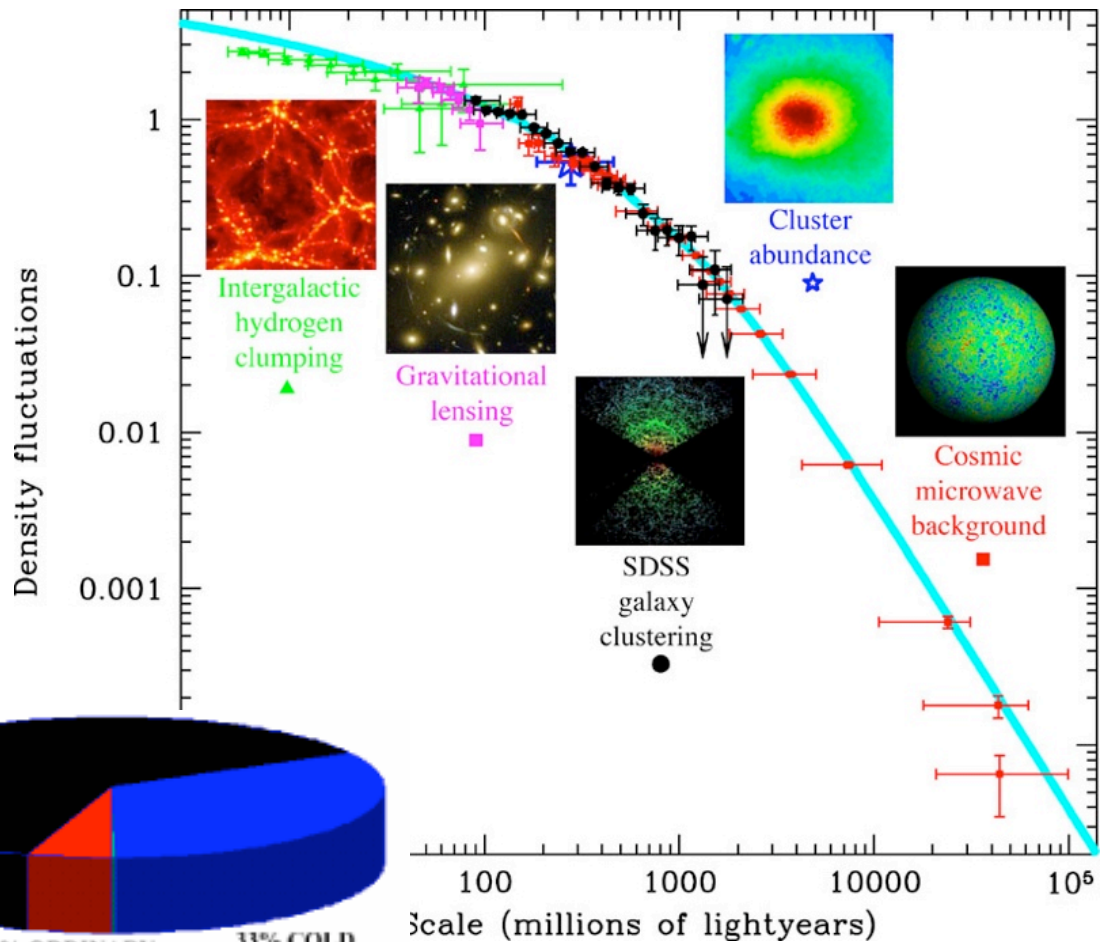
compute how the Universe came into existence  
follow how quantum fluctuations generated perturbations

1. follow how structures form in the dark matter
2. how gas accretes in these structures
3. and forms stars
4. and forms metals
5. and forms planets
6. and forms people
7. and .....

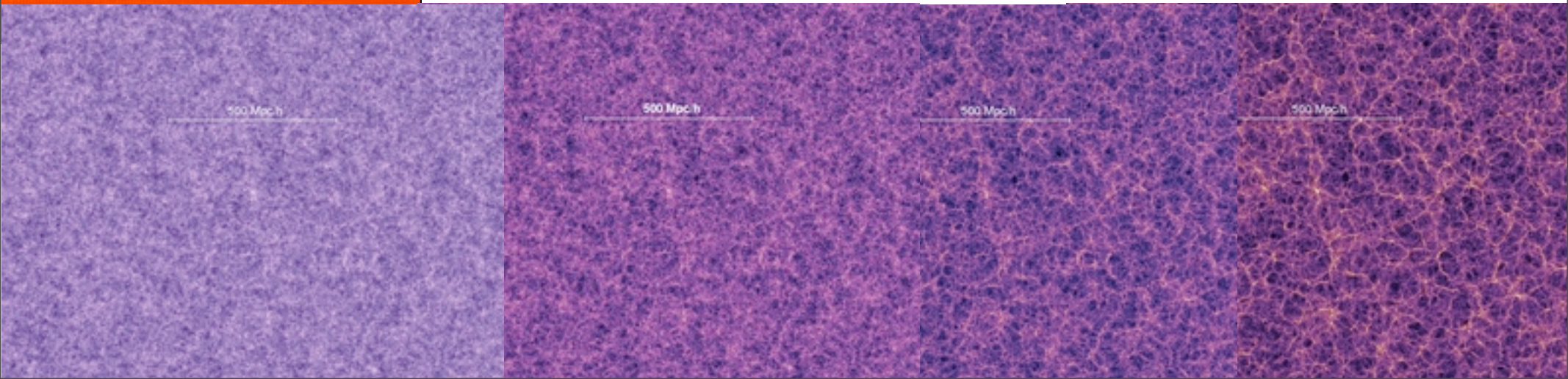
100001. and write some papers  
100002. and get a PhD



WMAP

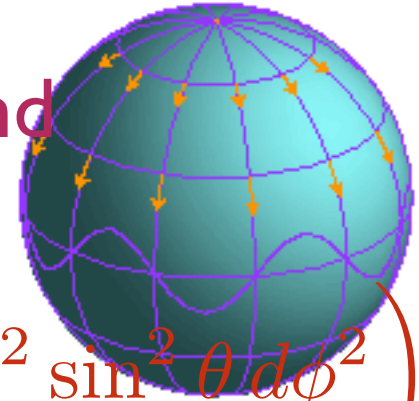


$t=0.0036$



# Basic description of space-time

1) Measuring distances in a homogeneous and isotropic Universe: FRW



$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

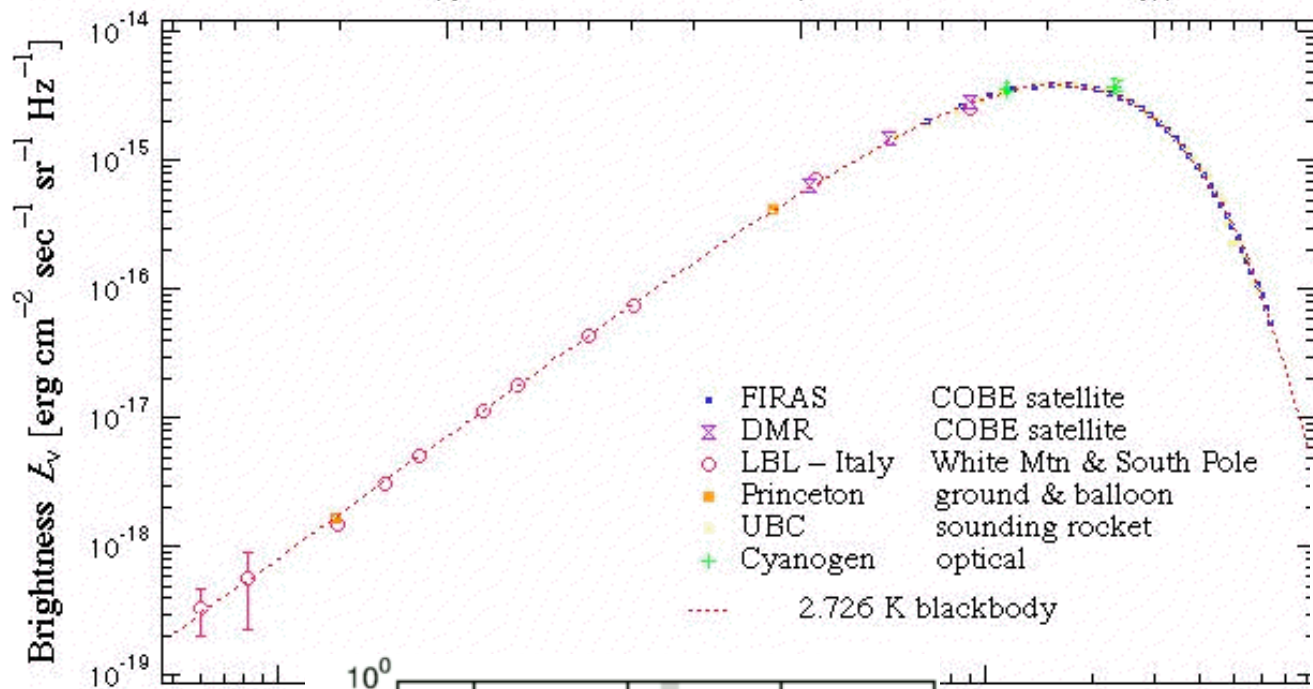
2) Dynamics of the expansion: GR

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

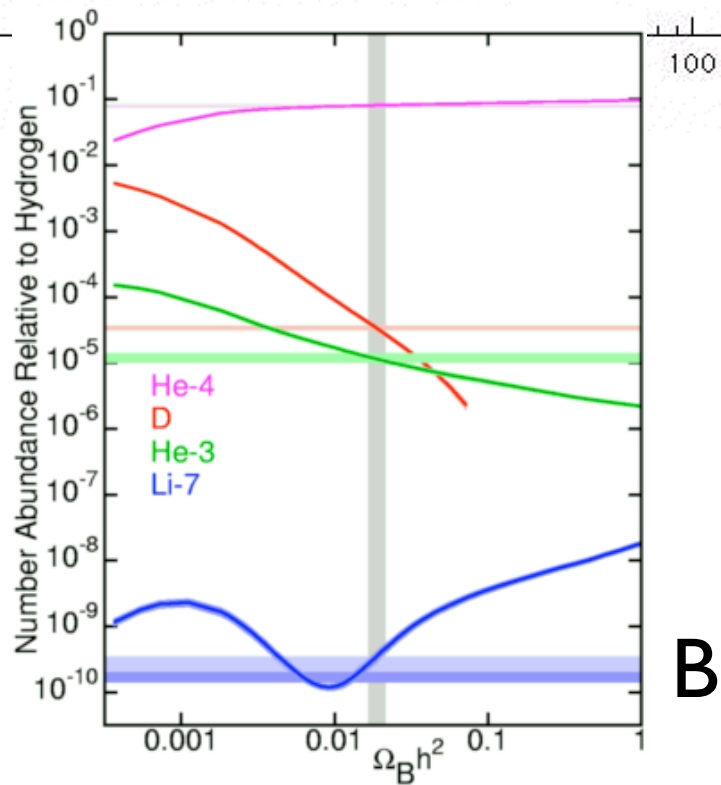
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda}{3}$$



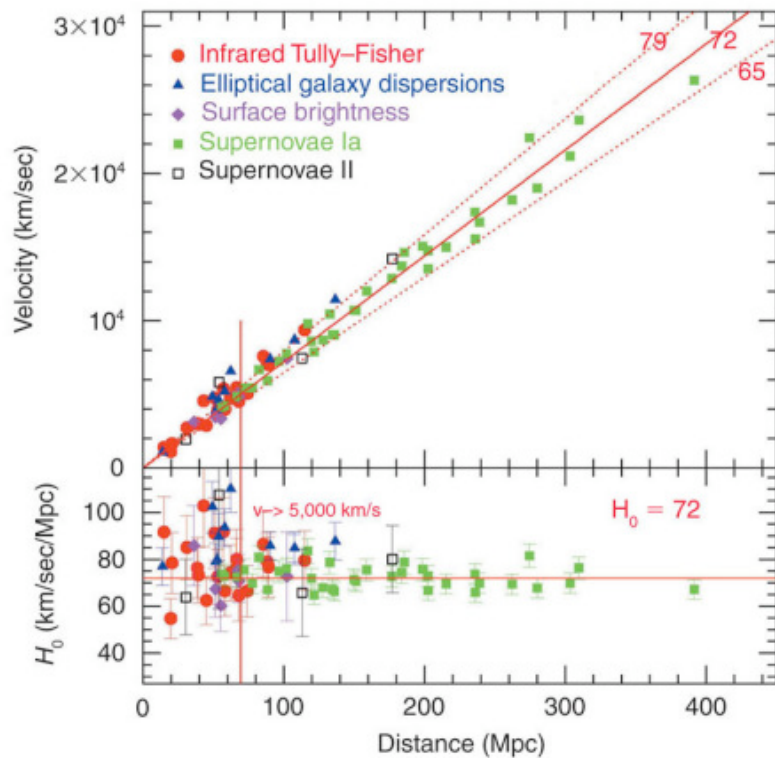
# The three pillars of this "Big-Bang" model



CMB



BBN



B  
(Wendy L. Freedman, Observatories of the Carnegie Institution of Washington, and NASA)

Expansion

## Growth of perturbations:

$$\frac{\delta \rho}{\delta t} + \nabla(\rho \mathbf{v}) = 0$$

$$\frac{\delta}{\delta t} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \phi$$

$$\rho \frac{\delta u}{\delta t} + \rho(\mathbf{v} \cdot \nabla) u = -p \nabla \mathbf{v}$$

$$\nabla^2 \phi = 4\pi G \rho$$

## Co-moving variables:

$$\mathbf{r} = a \mathbf{x}$$

$$\mathbf{v} = \dot{a} \mathbf{x} + a \dot{\mathbf{x}}$$

Physical values



Co-moving values

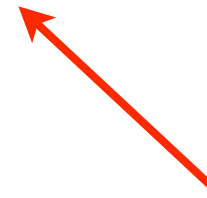


# Lagrangian versus Eulerian dynamics:

$$\left(\frac{\delta}{\delta t} + \mathbf{v} \cdot \nabla\right)\rho \equiv \frac{d\rho}{dt} = -\rho\nabla\mathbf{v}$$



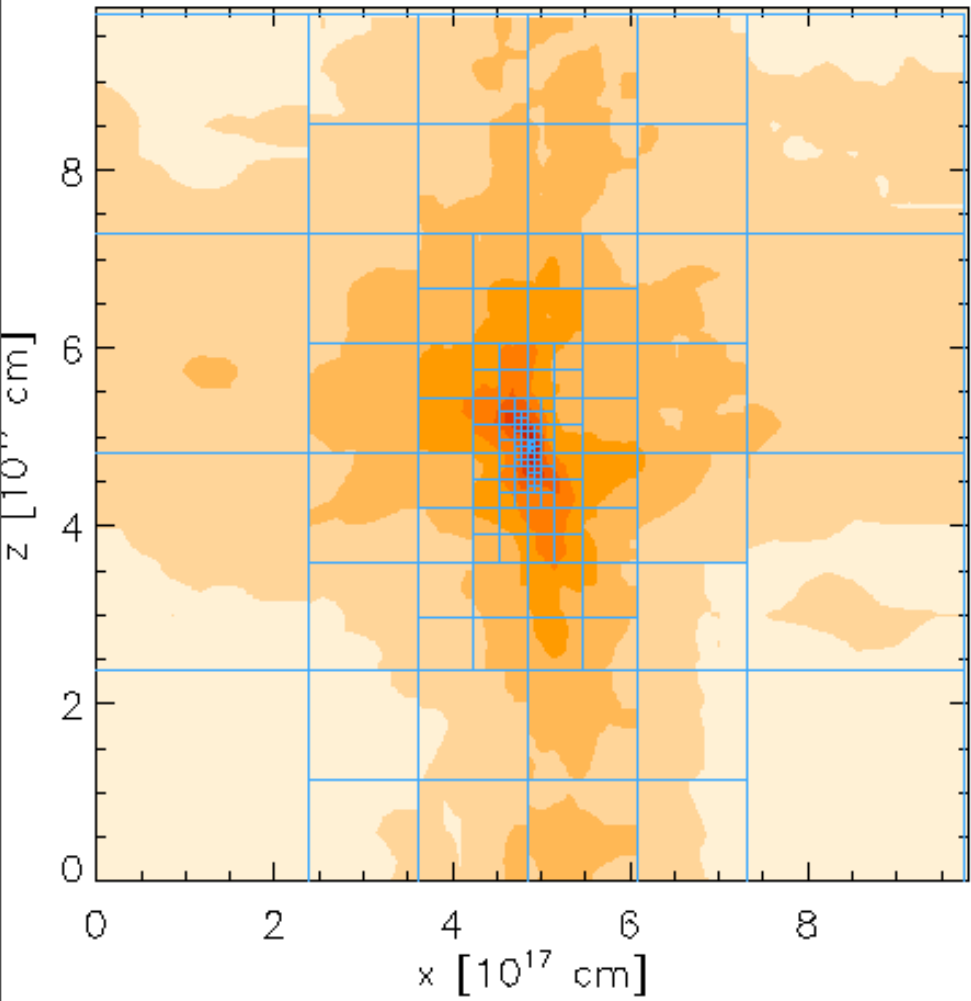
Eulerian time derivative



Lagrangian time derivative



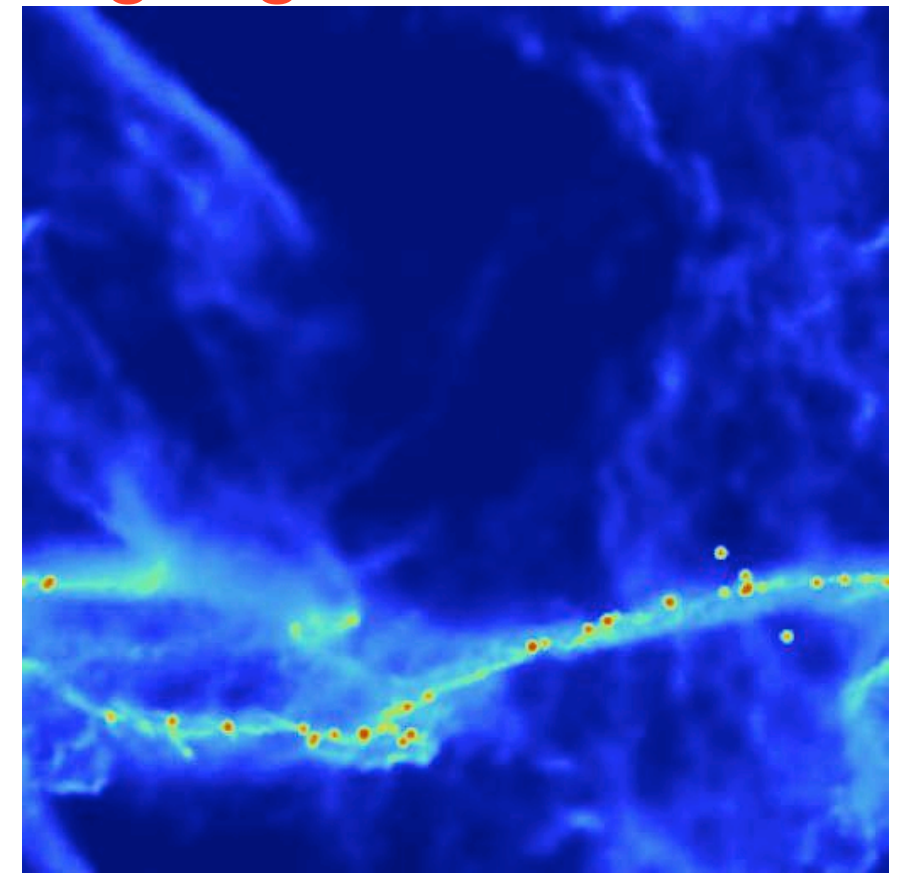
## Eulerian codes



**Zeus, Ramses, Flash,  
Enzo**

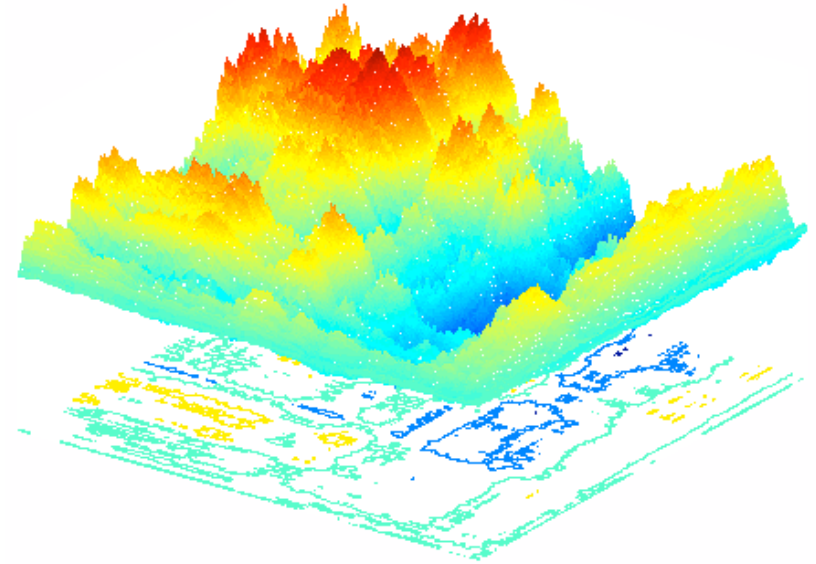
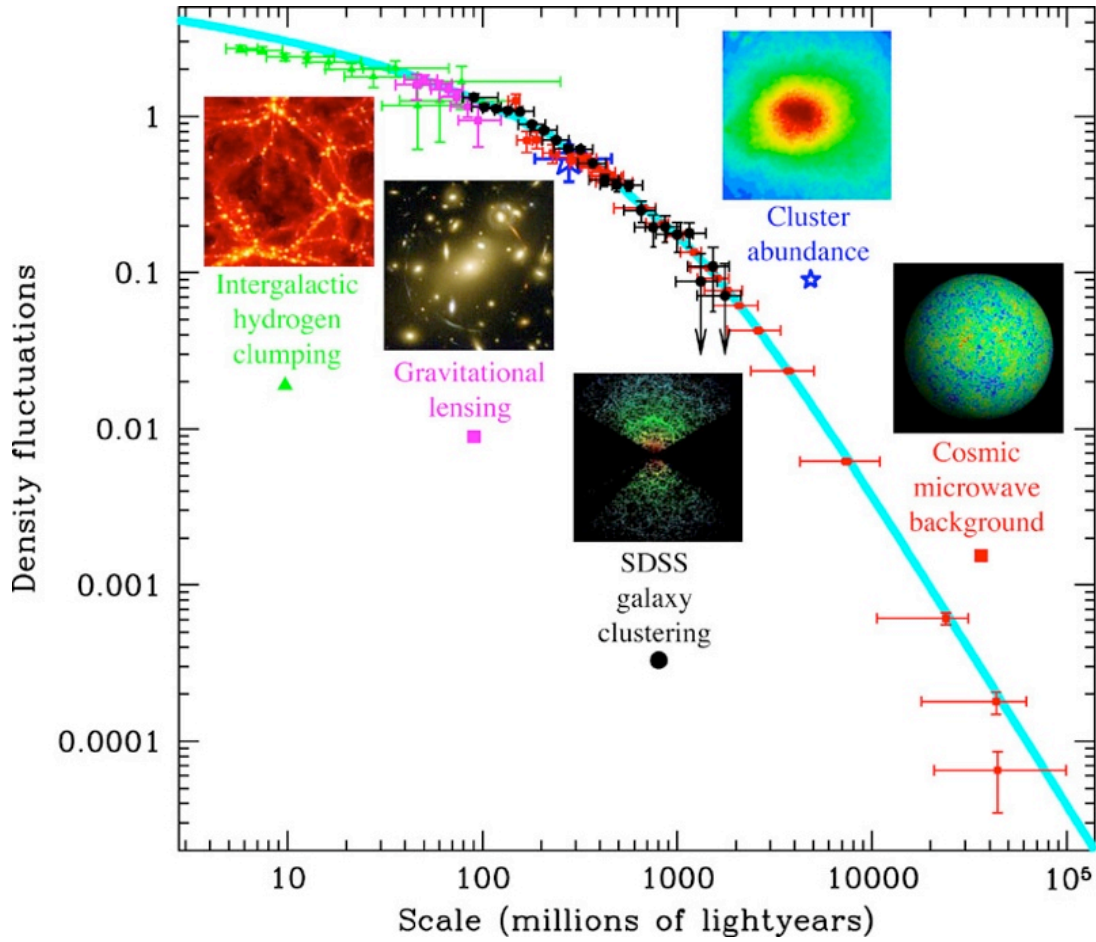
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## Lagrangian codes

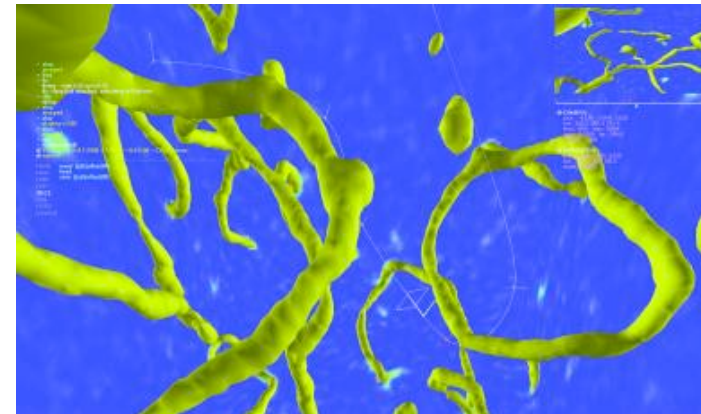


**Gadget, Hydra,  
Gasoline, TreeSPH**

# Initial conditions



Linear transfer function



## Numerical issues: time integration

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{a}$$

$$\mathbf{r}^{n+1} = \mathbf{r}^n + \mathbf{v}^n \Delta t + \frac{1}{2} \mathbf{a}^n \Delta t^2 + \dots$$

$$\frac{\mathbf{r}^{n+1} - \mathbf{r}^n}{\Delta t} = \mathbf{v}^n$$

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t} = \mathbf{a}^n$$

$$t = n\Delta t$$

$$\mathbf{r}^{n+1} = \mathbf{r}^n + \mathbf{v}^n \Delta t$$

$$\mathbf{r}^{n+1} = \mathbf{r}^n + \mathbf{v}^n \Delta t + \frac{1}{2} \mathbf{a}^n \Delta t^2 + \dots$$

$$\frac{\mathbf{r}^{n+1} - \mathbf{r}^n}{\Delta t} = \mathbf{v}^n$$

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t} = \mathbf{a}^n$$

$$t = n\Delta t$$

$$\mathbf{r}^{n+1} = \mathbf{r}^n + \mathbf{v}^n \Delta t$$

Euler scheme

$$\frac{\mathbf{r}^{n+1} - \mathbf{r}^n}{\Delta t} = \mathbf{v}^{n+1/2}$$

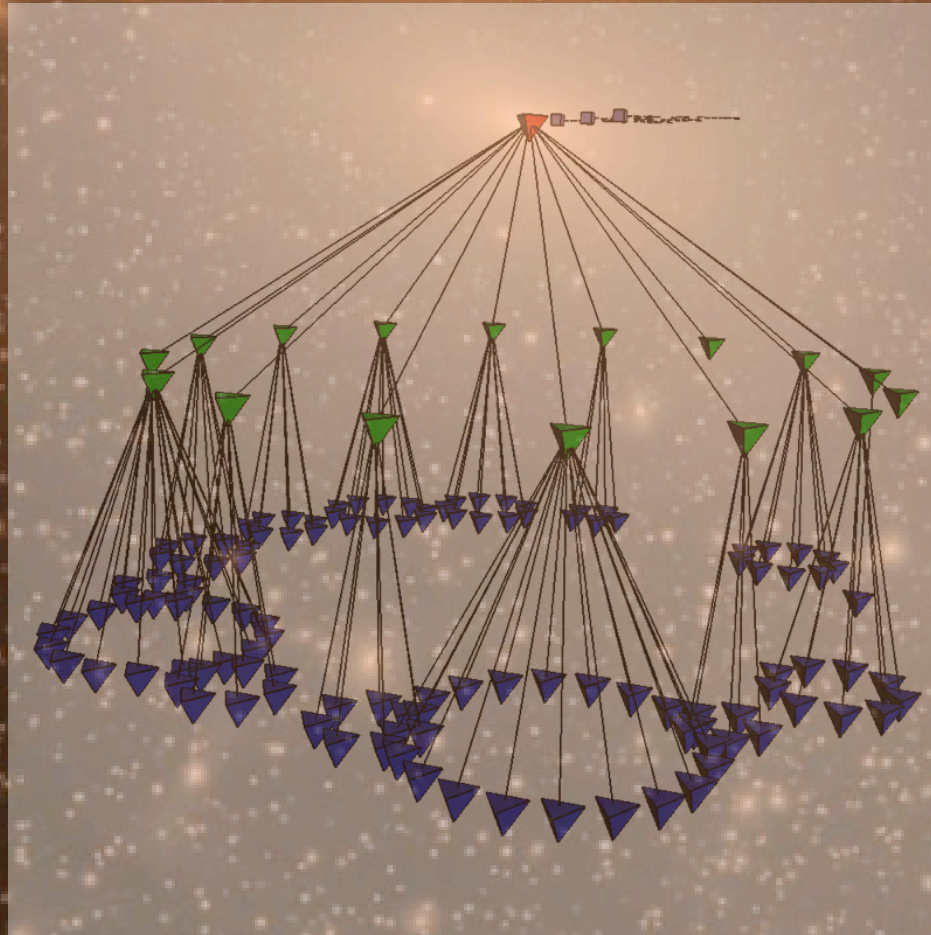
$$\frac{\mathbf{v}^{n+1/2} - \mathbf{v}^{n-1/2}}{\Delta t} = \mathbf{a}^n$$

Leap-frog scheme

$$\mathbf{r}^{n+1} = \mathbf{r}^n + \mathbf{v}^{n+1/2} \Delta t = \mathbf{r}^n + \mathbf{v}^n \Delta t + \frac{1}{2} \mathbf{a}^n \Delta t^2$$

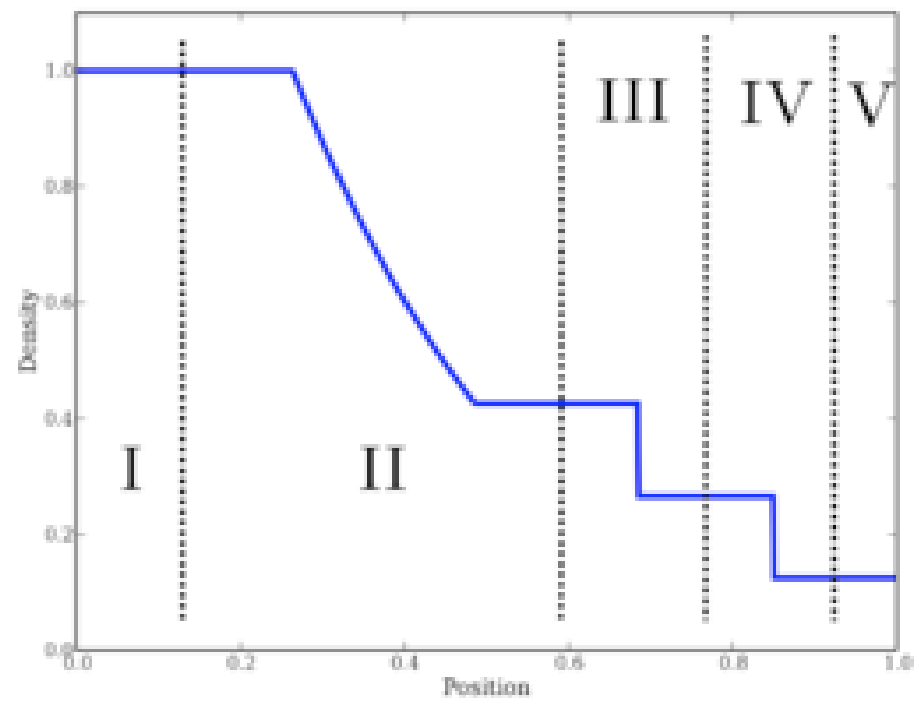
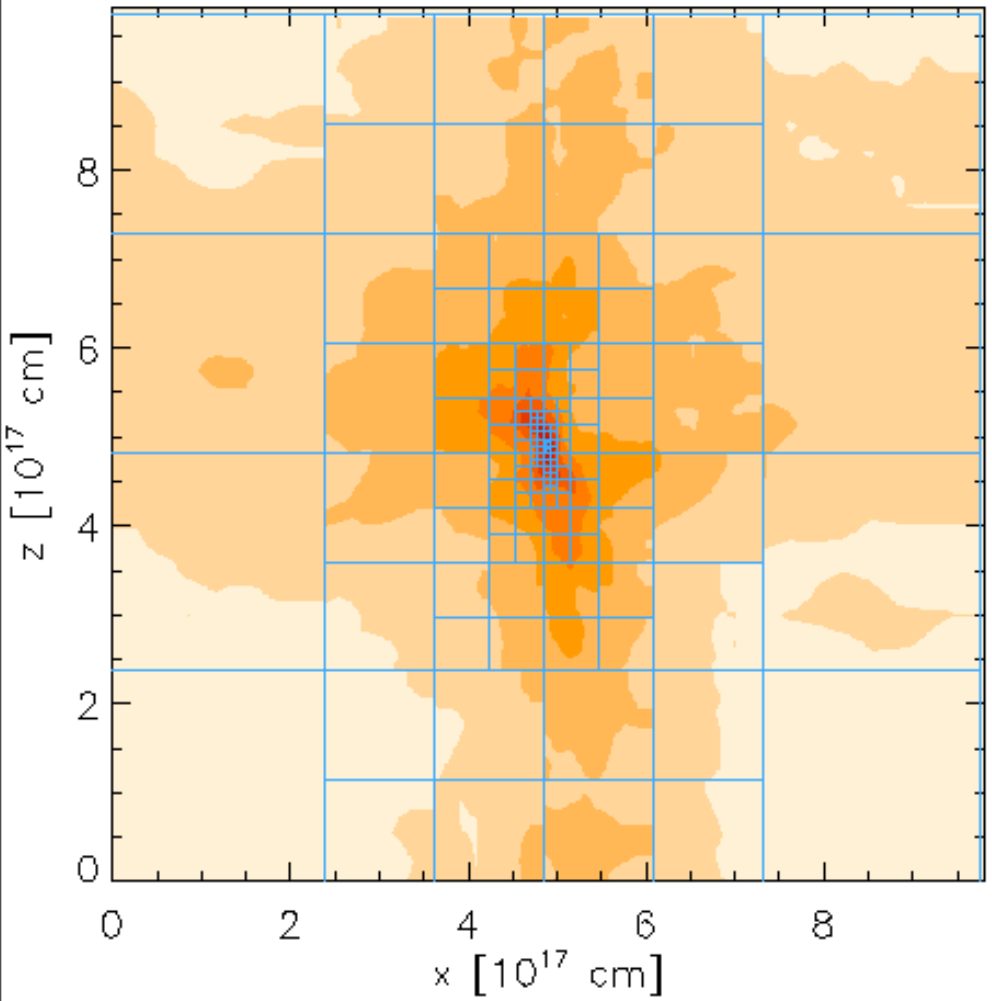
# Numerical issues: computing gravity

$$\mathbf{a}_i = \sum_{j \neq i} \frac{\mathbf{r}_i - \mathbf{r}_j}{r_{ij}^3} G m_j (\mathbf{r}_i - \mathbf{r}_j)$$

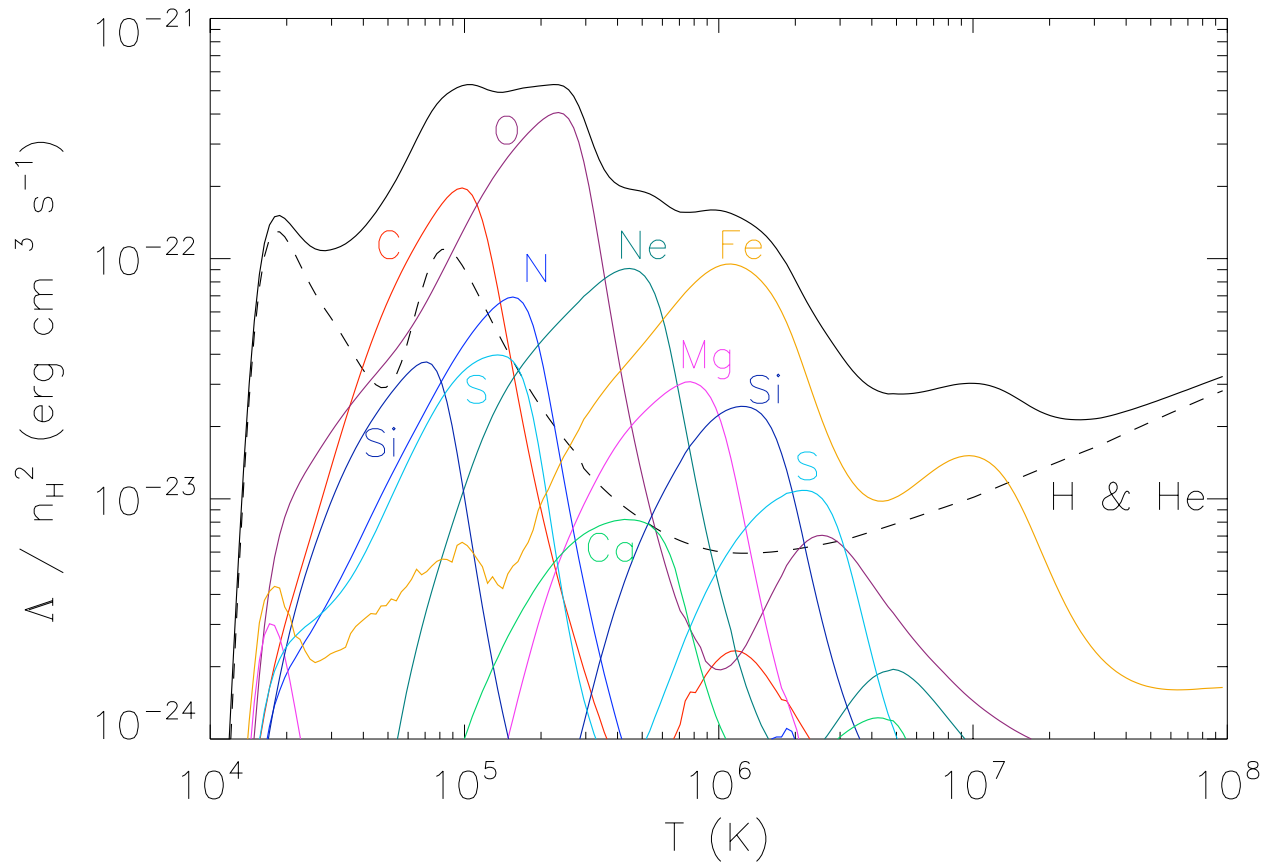


80 kpc

# Numerical issues: hydrodynamics



# Numerical issues: extra physics



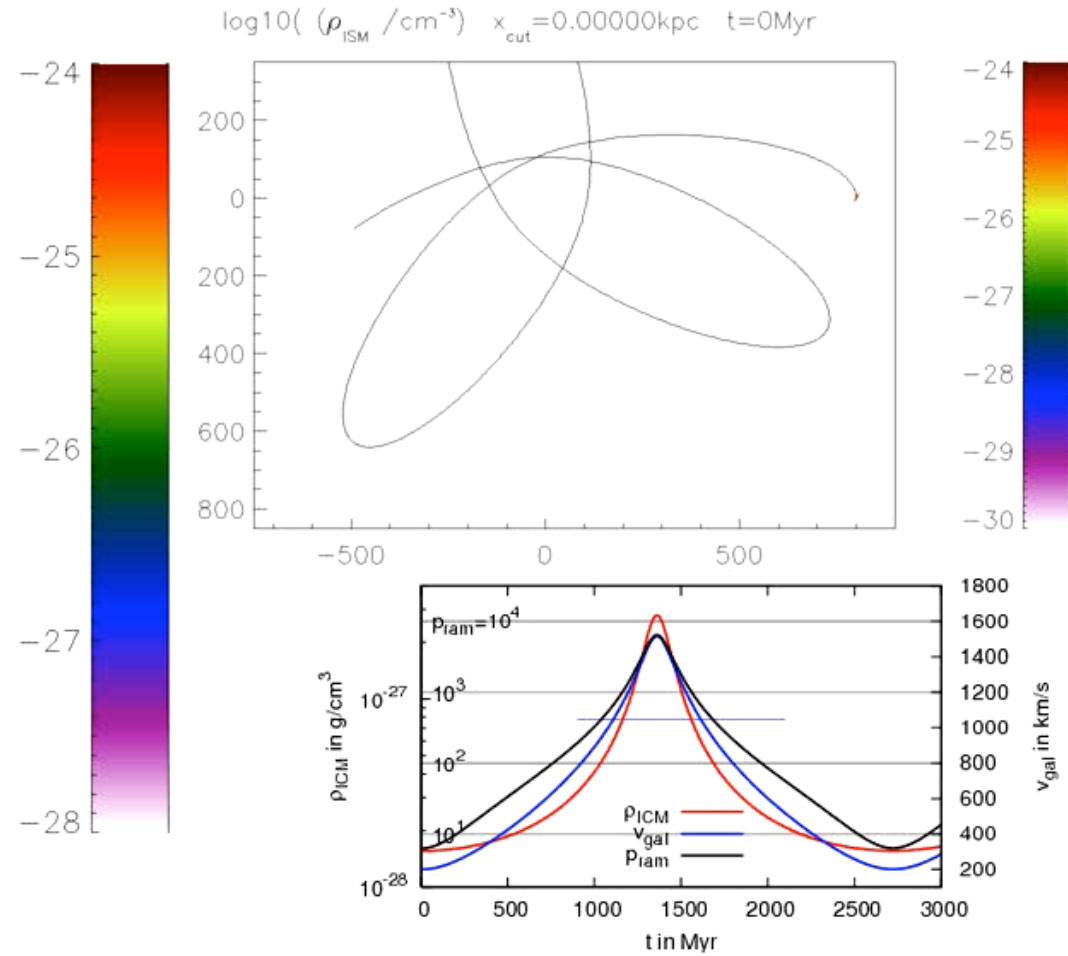
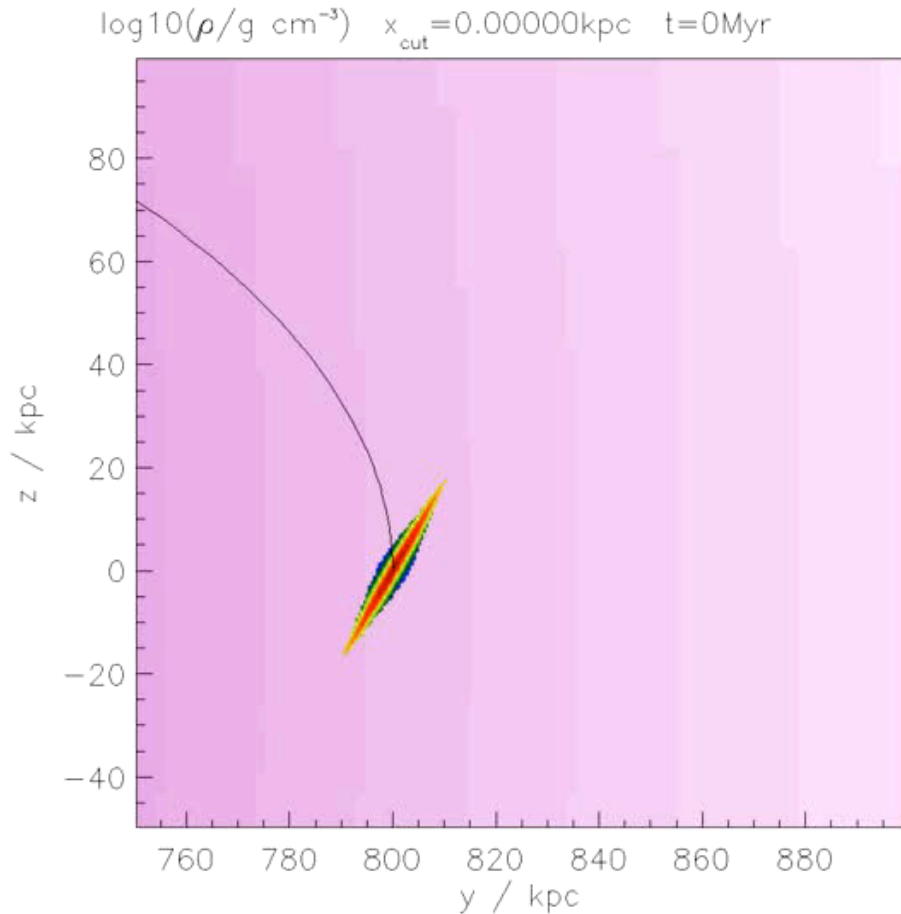
Radiative cooling rate of gas as function of temperature for different elements

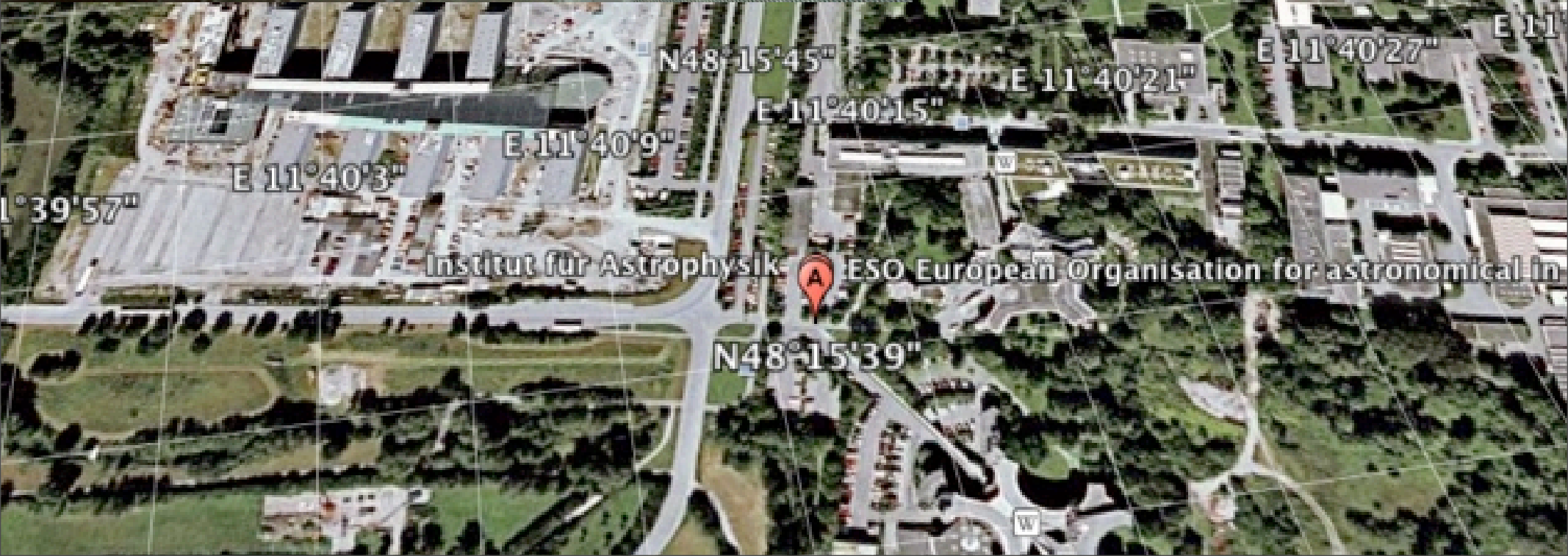
## Numerical issues: extra physics

- Magnetic fields
- Cosmic rays
- Energy input from stars
- Energy input from AGN
- Heating by ambient radiation field
- Cooling by gas, molecules, dust
- ....

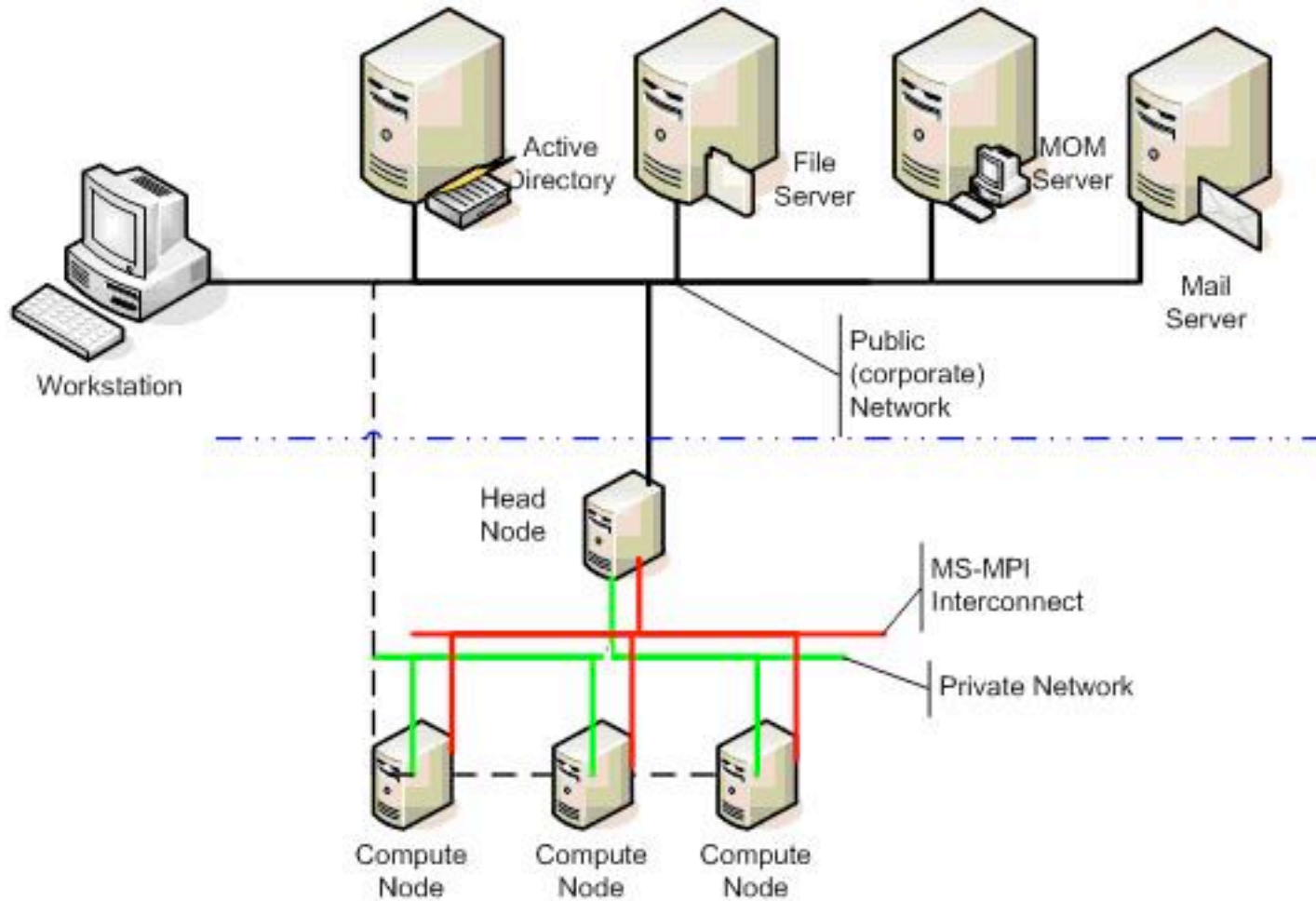
Role of AGN:  
Cold accretion  
during mergers  
gives rise to QSOs

# Numerical issues: extra physics





# Numerical issues: parallel computing





M Norman



V Springel



ENZO

C/C++/Fortran, 80 k lines)

Gadget

C, 80k lines

Flash

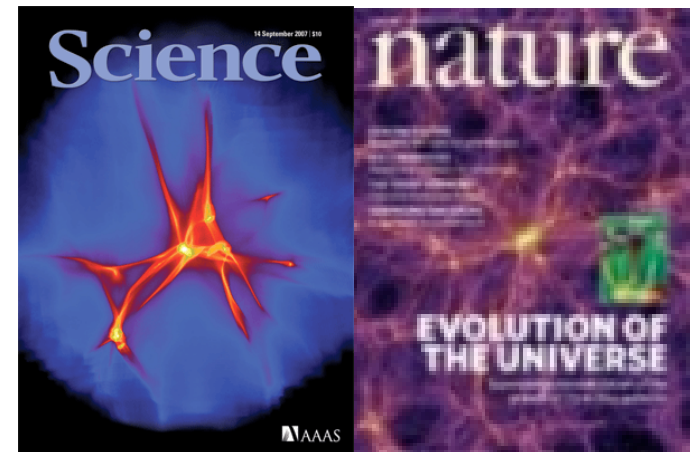
Fortran/C, 120k  
lines

## Writing simulation code requires man-years of effort

- Large simulations produce TBytes of data: analysis is major part of project
- Hard/impossible to analyse on small computer
- Big team to get most out of investment

Movies essential, not just for showing off, but as way of data-mining: discover unexpected features of simulation)

Data format, units and dimensions  
major sources of errors/hassle



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- That's cool! (some movies)
- That's hard!! (some equations)
- Where do we start? (ICs)
- How do we go from here? (integrators)
- Analysis

