University of Durham

EXAMINATION PAPER

May/June 2003 042541/01

Examination for the Degrees of B.Sc. and M.Sci.

LEVEL 2 PHYSICS: STARS AND GALAXIES

SECTION A. COMPULSORY QUESTION
SECTION B. OBSERVATIONAL TECHNIQUES
SECTION C. STARS
SECTION D. GALACTIC ASTRONOMY

Time allowed : $2\frac{1}{2}$ hours

Attempt **all parts** of question 1 and **two** questions selected from sections B, C and D. The marks shown in brackets for the main parts of each question are given as a guide to the weighting that the markers expect to apply. Question 1 carries 60% of the total marks for the paper.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

APPROVED TYPES OF CALCULATOR MAY BE USED.

Information

$e = 1.60 \times 10^{-19}$ C

$k_B = 1.38 \times 10^{-23}$ J K^{-1}

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$h = 6.63 \times 10^{-34}$ J s

Bohr magneton $= 9.27 \times 10^{-24}$ J T^{-1}

Nuclear magneton $= 5.05 \times 10^{-27}$ J T^{-1}

Avogadro's Constant $= 6.02 \times 10^{26}$ kmol^{-1}

Stefan-Boltzmann Constant $= 5.67 \times 10^{-8}$ W m^{-2} K^{-4}

$c = 3.00 \times 10^8$ m s^{-1}

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$\epsilon_o = 8.85 \times 10^{-12}$ F m^{-1}

$\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

$R = 8.31 \times 10^3$ J K^{-1} kmol^{-1}

$g = 9.81$ m s^{-2}

SECTION A. COMPULSORY QUESTION

Attempt **all** parts of Question 1.

1. (a) What are the two major types of telescope mount? State a relative advantage of each. [2 marks]

   (b) Write down the reflection grating equation of a spectrograph. Calculate the difference in angle for light reflected from a grating for two wavelengths of 550 nm and 560 nm. Assume that the grating is used in the first order, the angle of incidence is 45°, and the ruling density is 1000 lines/mm. [2 marks]

   (c) State the equation of hydrostatic equilibrium as used in stellar structure calculations. What boundary condition is generally applied when solving this? [2 marks]

   (d) The mean free path for a photon inside the Sun is roughly 0.01 m, and the time taken for absorption and re-emission for a photon is 10^{-8} sec. How long does it take for energy generated in the centre of the Sun to escape? [The radius of the Sun is 7×10^{8} m.] [2 marks]

   (e) What process provides the main support against gravity in the centres of white dwarf stars? Give an approximate value for the maximum mass (in units of solar masses) that such a star can have. [2 marks]

   (f) What defines the characteristic radius of a black hole? Calculate this radius for a black hole of 3 times the mass of the Sun. [The mass of the Sun is 2×10^{30} kg.] [2 marks]

   (g) What is the most common engine for radial pulsations in stars? Given that the Period-Luminosity relationship for Cepheid variables is:

   $$M = -2.8 \log_{10}(\Pi) - 1.43$$

   where M is the absolute magnitude and Π is the period in days, calculate the distance from the Earth in parsecs of a Cepheid variable star with an apparent magnitude of 6.05 and a period of 8 days. [2 marks]

   (h) State the formula for a pulsar's age, assuming it loses energy by magnetic dipole radiation. What is the estimated age in years for the Crab Nebula given its period of 0.0333 sec and period derivative of 4.21×10^{-13} sec sec^{-1}. [2 marks]

   (i) Two stars are observed to be orbiting each other, with the maximum radial velocity of star A being 10 km s^{-1} and that of star B being 40 km s^{-1}. What is the ratio of the mass of star A to that of star B? [2 marks]

   (j) An unusual X-ray source has been recently observed in a nearby galaxy that seems to have a luminosity of approximately 1.3 x 10^{34} W. If its luminosity is limited by the Eddington luminosity, what is its minimum mass? [Thompson cross section, $\sigma_T = 6.65 \times 10^{-29}$ m^2] [2 marks]

(k) What are the two main types of galaxies? Compare and contrast three important aspects in which they differ. [2 marks]

(l) What is meant by the rotation curve of a galaxy? Why do measured rotation curves suggest the presence of dark matter in the outer parts of disk galaxies? [2 marks]

(m) Oort's relations for the radial (v_r) and tangential (v_t) velocities of stars in a differentially rotating disk, are

$$v_r = A\,d\,\sin(2l) \;\text{ and }\; v_t = d\,(A\cos(2l) + B),$$

where d and l are the distance to the star and the galactic longitude respectively. A star is seen at 0.5 kpc in the direction of the galactic centre. Compute the radial and tangential velocity of the star. How much is the proper motion in arcseconds per year? Adopt $A = 14$ km s^{-1} kpc^{-1} and $B = -12$ km s^{-1} kpc^{-1} for Oort's constants. [1pc $= 3.09\times 10^{16}$m] [2 marks]

(n) Describe two key observations that suggest the presence of super massive black holes in the centres of galaxies. [2 marks]

(o) Very briefly describe how the motions of the galaxies, and gravitational lensing, have been used to estimate the mass of galaxy clusters. [2 marks]

SECTION B. OBSERVATIONAL TECHNIQUES

2. Write down an equation for the signal to noise ratio of an observation in terms of the detected signal and detected sky background. Assume that the detector is noiseless. [2 marks]

   Express this equation in terms of the photon flux at the top of the atmosphere from the star F_s, the photon flux from the background F_{bg}, the transmission of the atmosphere ϵ_a, the detector's quantum efficiency ϵ_d, the size of the primary mirror D, the bandwidth $\Delta\lambda$ and the exposure time t. [4 marks]

   You have the choice of observing at two sites with identical telescopes and detectors. The atmosphere at site A has a transparency which is 1.2 times the value at site B. However the sky background (due to water vapour) is 20% higher at site A than site B. By calculating the relative sizes of the two signal to noise ratios, determine which is the best site. [3 marks]

   Describe qualitatively, the effect of having a noisy detector on the above calculation. [1 mark]

SECTION C. STARS

3. Describe the two main factors that determine the probability of a nuclear reaction occurring in the Sun. [2 marks]

   Calculate the ratio of the typical particle kinetic energy to the height of the Coulomb barrier that particles have to surmount in order to get within the range of strong interactions, given that a typical temperature at the centre of the Sun is around 10^7 K and the range of the strong force is comparable to nuclear dimensions, i.e. 10^{-15} m. [3 marks]

   Assuming that the PPI chain produces the entire energy output from the Sun, where the energy release per helium nucleus formed is approximately 27 MeV, calculate the rate of solar neutrinos passing through the top of your head during this exam. Explain qualitatively why this flux should have little or no effect on your exam performance. [4 marks]

   How would your answer change if you were taking the exam while orbiting a white dwarf star with the same mass as the Sun? [1 mark]

   [Take the luminosity of the Sun to be 4×10^{26} W, 1 MeV $= 1.6 \times 10^{-13}$ J, 1 astronomical unit $= 1.5 \times 10^{11}$ m]

4. Sketch the Hertzsprung-Russell diagram to show the main areas occupied by radially pulsating stars in relation to the main sequence. What properties must the opacity have for pulsations to occur, and in what regions can this happen? [4 marks]

   The period of non-radial pulsations on the surface of the Sun is the time taken for sound to travel one horizontal wavelength at the adiabatic sound speed, and the horizontal wavelength for non-radial mode, l, is given by the circumference of the sun divided by $\sqrt{l(l+1)}$. Calculate the temperature of the solar surface, given that the $l = 150$ non-radial pulsation of the Sun has a period of 38.6 minutes. You may assume that the plasma at the surface of the Sun behaves like a perfect gas with a mean molecular weight, $\mu = 0.5$, and "the ratio of specific heats", $\gamma = 5/3$. [4 marks]

   Check this temperature by calculating the predicted luminosity of the Sun by assuming it is a perfect black body radiating at this temperature. Does the answer seem reasonable? [2 marks]

   [The radius of the Sun is 7×10^8 m. Take the luminosity of the Sun to be 4×10^{26} W.]

SECTION D. GALACTIC ASTRONOMY

5. The expression for the Jeans mass M_J in a gas of temperature T and mass density ρ is

$$M_J = \left(\frac{3k_B T}{2m_p G}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} .$$

Explain the concept of Jeans mass. [1 mark]

Suppose that due to the passage of a spiral arm, small clouds merge into larger clouds. Explain why you expect star formation to start in clouds with masses of order of the Jeans mass. [1 mark]

Suppose that the radius of a spherical cloud, with mass slightly above the Jeans mass, has changed from R to αR when collapsing. Assuming the change in temperature follows $T \propto \rho^{\gamma-1}$, compute the ratio of the new to the old Jeans mass. [2 marks]

Evaluate this ratio assuming $\alpha = 0.5$, for both $\gamma = 5/3$ and $\gamma = 1$. Interpret the result in terms of whether you expect the cloud to fragment on collapse. [2 marks]

To estimate the collapse time of the cloud, consider the following model. A test particle is released from rest, at distance r from the centre of a spherical, uniform density sphere, with density ρ. Assuming ρ remains constant, show that the equation of motion is $d^2r/dt^2 = -(4\pi G\rho/3)r$. [2 marks]

For a pure hydrogen gas, with a density of 100 hydrogen atoms per cm^{-3}, find the collapse time T_c when the particle reaches the centre. [2 marks]