1 Stars & Galaxies, Galaxies example class 2, answers

Question 1.

1. Rotation curve is the circular velocity, V_c, as function of radius, R. On a circular orbit, $V_c^2 = GM(< R)/R$. For Keplerian motion, $M(< R) = M$ is a constant, hence $V_c = \sqrt{GM/R}$.

2. Measurements of Oort's constants, obtained from observations of the motions of stars near the sun, yield $dV_c/dR \approx 0$, implying V_c constant. Also HI 21-cm measurements show flat rotation curves.

Question 2.

1. Take derivative with respect to R of $R\,V_c^2 = GM(< R)$ yields $\rho(R) = V_c^2/4\pi\,GR^2$, so $\rho(R) \propto 1/R^2$.

2. Substitute the numbers in previous expression yields $\rho(R = 8.5\,\mathrm{kpc}) = 0.012\,M_\odot\,\mathrm{pc}^{-3} = 8.4 \times 10^{-22}$ kg m^{-3}.

3. From above the ratio dark matter over stars is 0.012.

Question 3.

1. Eq.(4.4) in the notes gives $R_S = ((3\dot{N})/(4\pi\alpha n^2))^{1/3} = 30$ pc for the Strömgren radius.

2. If R is the current position of the ionisation front, then there are $(4\pi/3)\alpha\,n^2R^3$ recombination per unit time. To advance R to $R + \dot{R}\,dt$ in a time dt requires ionsing another $4\pi\,nR^2\dot{R}\,dt$ neutral atoms. Equating the number of required ionisations per unit time, $(4\pi/3)\alpha\,n^2R^3 + 4\pi\,nR^2\dot{R}$, with the number of photons emitted by the source, $\dot{N}$, yields the required relation.

3. The derivative is

$$
\begin{aligned}
\dot{R} &= \frac{R_0}{3t_0}\exp(-t/t_0)/(1-\exp(-t/t_0))^{2/3} \\
&= \frac{R_0}{3t_0}\,\frac{1-(1-\exp(-t/t_0))}{(R/R_0)^2} \\
&= -\frac{1}{3t_0}\,R + \frac{R_0^3}{3t_0}\,\frac{1}{R^2} \\
&= -\frac{\alpha n}{3}\,R + \frac{\dot{N}}{4\pi n}\,\frac{1}{R^2}\,. \qquad (1)
\end{aligned}
$$

Comparing the last two lines shows that the solution requires

$$
\begin{aligned}
t_c &= \frac{1}{\alpha n} \\
R_0^3 &= \frac{3\dot{N}}{4\pi\alpha n^2}\,. \qquad (2)
\end{aligned}
$$

It shows that R_0 is the Strögren radius. For small times, $t \ll t_0$, the solution is $(R/R_0)^3 = (t/t_0)$, hence the speed of the front is $\dot{R} = (R_0^3/3R^2)\,1/t_0 = \dot{N}/4\pi\,nR^2$