Stars & galaxies, multiple choice questions

Problem 1.

1. Answer **a**: The distance $r = 300$ kpc is beyond the extent of the dark halo (which has radius 200 kpc), hence equate $v_c^2 = GM/r$, where $M = 1.0 \times 10^{12} M_\odot$ is the total mass of the halo, to find $v_c = 170$ km s^{-1}

2. Answer **b**: The $1/r^2$ density profile gives rise to a flat rotation curve inside the halo. Therefore the circular velocity at $r = 100$ kpc is the same as at $r = 200$ kpc where it is $v_h^2 = GM/r$, with $M = 2.0 \times 10^{12} M_\odot$, hence $v_h = 207$km s^{-1}.

3. Answer **c**: The circular velocity is due to dark halo and bulge combined, so $v_c^2 = GM_{\rm halo}(< r)/r + GM_{\rm bulge}(< r)/r = v_h^2 + GM_{\rm bulge}(< r)/r$. Since the bulge is uniform density, $M_{\rm bulge}(< r) = (r/r_b)^3 M_b$, hence $M_b(r < 2{\rm kpc}) = (2/4)^3 M_b$, hence $v_c = 220$ km s^{-1}

4. Answer **b**: The expression for Oort's constant A is given by Eq. (5.5) in the notes,

$$\begin{aligned} A &= -\frac{1}{2}\left(\frac{dV_c}{dr} - \frac{v_c}{r}\right) \\ &= -\frac{1}{2}\left(\frac{GM_b}{r_b^3}\,\frac{r}{v_c} - \frac{v_c}{r}\right) \\ &= 48.9 \text{ km s}^{-1}\,, \end{aligned}$$

   with $v_c = 220$ km s^{-1}from above

Problem 2.

1. Answer **b**: The 3D velocity dispersion $\sigma_{\rm 3D}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$ which, if the orbits are isotropic, $\sigma_{\rm 3D}^2 = 3\sigma_{1D}^2$, hence $\sigma_{\rm 3D} = 1386$km s^{-1}.

2. Answer **b**: The virial mass is given by Eq. (9.4) in the notes,

$$M = \frac{R\sigma_{\rm 3D}^2}{G} = 2.2 \times 10^{14} M_\odot\,. \qquad (1)$$

3. Answer **a**: The virial temperature is given by Eq. (9.7) in the notes,

$$T = \frac{2\mu m_p}{3\mathrm{k_B}} \frac{GM}{R} = 1.7 \times 10^8 \,\mathrm{K}\,, \tag{2}$$

where the mean molecular weight per particle, $\mu = 0.5$, if we assume fully ionised hydrogen. At this temperature, the typical kinetic energy of a particle, $\mathrm{k_B}T \sim 15$ keV, is much large than the binding energy of Hydrogen, 13.6 eV, which is why the gas is indeed highly ionised.

4. Answer **b**: Given mass and radius, we can compute the 3D, and hence 1D velocity dispersion of the cluster, $\sigma_{\rm 1D} = \sigma_{\rm 3D}/\sqrt{3}$, with $\sigma^2_{\rm 3D} = GM/R$. Then the crossing time is

$$t_{\rm cr} = \frac{R}{\sigma_{\rm 1D}} = 820\,\mathrm{Myr}. \tag{3}$$