Stars & galaxies, example class 2

Question 1. Short questions

1. What is the 'rotation curve' of a spiral galaxy? Derive the expression for the rotation curve in a Keplerian disk.

2. Describe two key observations that suggest the presence of dark matter in the Milky Way.

Question 2. Galaxy rotation curves

1. Show that the density profile that correspondents to a flat rotation curve is $\rho(r) \propto 1/r^2$.

2. The circular velocity for the Milky Way is $V_c = 220\text{km s}^{-1}$. Compute the (dark) matter density at the position of the Sun, $r = 8.5\text{kpc}$.

3. The density of stars in the solar neighbourhood is around $n_0 = 1\ \text{pc}^{-3}$. Compute the ratio of dark matter to stars near the sun, assuming all stars have mass 1 $M_\odot$.

Question 3. HII regions

1. A gas cloud has uniform density of 1 hydrogen atoms per cm^3. A central O stars emits $\dot{N} = 10^{48}$ ionising photons per second. Compute the Strömgren radius in this cloud. Use $\alpha = 3.1 \times 10^{-13}\ \text{cm}^{-3}\ \text{s}^{-1}$ for the recombination coefficient.

2. In the discussion of the HII region we neglected recombinations. Demonstrate that recombinations change Eq. (4.2) to

$$\dot{N} = 4\pi R^2\, n\, \dot{R} + (4\pi/3)\, \alpha\, n^2\, R^3\,.$$

3. Show that $R = R_0\,(1 - \exp(-t/t_0))^{1/3}$ is the general solution to this equation. Show it reverts to the solution in Eq.(4.2) for $t \ll t_0$. Obtain expressions for t_0 and R_0.

$1\ \text{pc} = 3.09 \times 10^{16}\ \text{m}$, $M_\odot = 2.0 \times 10^{30}\ \text{kg}$, $G = 6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$