Stars & galaxies, example class 4

Short questions

1. Discuss three lines of evidence that suggest AGN are powered by a super-massive black hole.

2. What is the observational evidence the the Milky Way contains a super-massive central black hole?

3. How can you use gravitational lensing to determine the mass of a cluster of galaxies?

Question 1.
The initial mass function (or IMF), $N(M)dM$, is the number of stars born with mass between M and $M + dM$. Assume stars are born with the IMF

$$N(M)dM = N_0 \left(M/M_\odot\right)^{-2.35} dM/M_\odot \tag{1}$$

for $0.1 < M/M_\odot < 30$.

- A star cluster has just formed $10^6 M_\odot$ solar masses of stars. Determine the proportionality constant, then determine the number of SNe stars that will go off. (Assume stars with mass $> 8M_\odot$ go super nova).

- The luminosity of main sequence stars depends strongly on mass. Assume this relation is $L/L_\odot = (M/M_\odot)^2$. Compute the mean mass-to-light ratio of the stars in the cluster.

- The massive stars in the cluster contribute a lot to the light, however these stars do not live for very long. Compute the mean mass-to-light ratio when the cluster is old, and all stars with mass $> 1M_\odot$ have died.

Question 2.

A $10^{13} M_\odot$ elliptical galaxy at distance 500Mpc, lenses a spiral galaxy a distance 500Mpc behind it. For exact alignment, compute the angular radius of the Einstein ring in arc seconds. Compare the radius of the elliptical, 20kpc, with the size of the ring.

Question 3.
The density distribution in a galaxy is

$$\begin{aligned} \rho(r) &= \rho_1 \frac{r_1}{r} \text{ for } r \leq r_1 \\ &= \rho_1 \left(\frac{r_1}{r}\right)^2 \text{ for } r_1 \leq r \leq r_2 \\ &= 0 \text{ for } r > r_2 \,. \end{aligned} \tag{2}$$

In its centre, it has a super massive black hole with mass $M_0 = 10^6 M_\odot$. Assume $r_1 = 1$kpc and $r_2 = 100$kpc.

- compute the total mass $M(r)$ enclosed by a sphere with radius r.
- compute the circular velocity $V_c(r)$.
- determine ρ_1 such that $V_c = 220\text{km s}^{-1}$ at $r = 8.5$kpc.
- for this value of ρ_1, sketch $V_c(r)$.
- suppose this galaxy is at a distance $d = 5$Mpc from us. To find dynamical evidence for the presence of the black hole, estimate the required angular resolution of the observations.