Stars & galaxies, weekly problems

Problem 2.

Short questions

1. What is the physical process that gives rise to the Hydrogen 21-cm line? How is the 21-cm line used to measure rotation curves of spiral galaxies? [2 marks]

2. What is the 'winding' problem of spiral galaxies? How is this problem resolved? [2 marks]

Long question

The speed, $V_c(R)$, of a particle on a circular orbit with radius R, enclosing a mass $M(< R)$ is $V_c^2(R) = GM(< R)/R$.

1. Demonstrate that a constant rotation speed, $V_c = V_{c,0}$, requires a density distribution $\rho(R) = V_{c,0}^2/4\pi GR^2$. Assume this is the dark matter distribution in the Milky Way. [2 marks]

2. The surface density of stars in the Milky Way's disk falls with distance R to the centre as $\sigma(R) = \sigma_0 \exp(-R/R_0)$. Derive an expression for the disk mass, $M_d(< R)$, enclosed in a sphere of radius R. [2 marks]

3. In the Milky Way galaxy, both the dark halo and the disk itself contribute to the rotation speed. Use your previous results to compute $V_c(R)$ at distance $R = 1$ kpc and $R = 10$ kpc. Assume $\sigma_0 = 2 \times 10^4\, M_\odot$ pc^{-2}, $R_0 = 0.3$ kpc and $V_{c,0} = 200$ km s^{-1}. [2 marks]

1 pc $= 3.09 \times 10^{16}$ m, $M_\odot = 2.0 \times 10^{30}$ kg, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}