Stars & galaxies, homework problems

Problem 3. Hydrostatic equilibrium of the hot gas and velocity dispersion of stars in an elliptical galaxy.

1. Explain how X-ray observations are used to determine the temperature, $T(r)$, and density, $\rho_{\rm gas}(r)$, of the hot gas in an elliptical galaxy. [2 marks]

2. X-ray analysis shows that T is constant, and $\rho_{\rm gas}(r) \propto 1/r^2$. Use the equation of hydrostatic equilibrium to show that the total mass density (including gas and dark matter) follows $\rho \propto 1/r^2$. Assume this density distribution in what follows. [2 marks]

3. Show that the enclosed mass $M(<r) = (2{\rm k_B}T/G\mu m_p)\, r$ and compute this for $r = 20$kpc. Assume the gas is fully ionised hydrogen with molecular weight $\mu = 0.5$ and temperature $T = 2 \times 10^7$K. [2 marks]

4. The equation for the isotropic velocity dispersion of the stars, $\sigma(r)$, is the Jeans equation,

$$\frac{GM(<r)}{r^2} = -\frac{1}{\rho}\frac{d}{dr}\left(\rho\sigma^2\right).$$

   Compute the constant velocity dispersion $\sigma(r) = \sigma_0$ that solves this equation. [2 marks]

5. Find the escape velocity for this galaxy at $r = 10$kpc, assuming the density drops to zero beyond $r_\star = 20$kpc. [2 marks]

1 pc $= 3.09 \times 10^{16}$ m, $M_\odot = 2.0 \times 10^{30}$ kg, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}