Stars & galaxies, weekly problems

Problem 2.

Short questions

1. A neutral hydrogen atoms contains a proton and an electron, both of which have a quantum mechanical property called spin. When these are parallel, the energy of the atom is higher than when they are anti-parallel. When the spin changes from parallel to anti-parallel, the atom emits a 21-cm photon. To measure a rotation curve, determine the wavelength λ of the 21-cm photon as function of distance to the centre R, and use the equation for Doppler shift, to derive speed from λ.

2. If a spiral arm were a material structure, it would wind-up very quickly because of the differential rotation of the disk, making all spiral structures very tightly wound, which is not observed. This means that spiral arms are not material structures, and stars move in and out of them.

Long question

1. Taking the derivative with respect to R of both sides of $RV_c^2 = G(M < R)$ yields $V_c^2 = G\,dM/dR = 4\pi\,G\rho(R)R^2$ when $V_c = V_{c,0}$ is constant. Re-arranging yields $\rho(r) = V_{c,0}^2/4\pi GR^2$

2. Using cylindrical coordinates, the enclosed mass $M_d(< R) = \int_0^R \sigma(R) 2\pi\, RdR = 2\pi\,\sigma_0\,R_0^2\,(1-(1+x)\exp(-x))$, where $x \equiv R/R_0$.

3. The rotation speed due to dark halo and disk combined, is $V_c^2(R) = V_{c,0}^2 + V_d^2(R)$, where $V_d^2(R) = GM_d(R)/R$ is the enclosed mass due to the disk. Substituting the numbers yields $V_c(R = 1\text{kpc}) = 280$ km s^{-1} and $V_c(R = 10\text{kpc}) = 210$ km s^{-1}.