# Stars & galaxies, homework problems

## Problem 3.

1. $T(r)$ is determined from modelling the line-strengths in the X-ray spectrum, $\rho(r, T)$ from the X-ray emissivity due to thermal bremsstrahlung, $\propto \rho^2 T^{1/2}$.

2. In hydrostatic equilibrium, $GM/r^2 = -(1/\rho_g)dp/dr$, where $p = (\mathrm{k_B}T/\mu m_p)\rho_g$ is the gas pressure and $\rho_g$ the gas density. This simplifies when $T$ is constant to $GM = -(\mathrm{k_B}T/\mu m_p)r^2 d\ln\rho_g/dr$. If $\rho_g \propto 1/r^2$, the rhs of this equation becomes $(2\mathrm{k_B}T/\mu m_p)r$, or $M(< r) \propto r$. Taking the derivative of both sides wrt $r$ shows $\rho_{\mathrm{tot}}(r) \propto 1/r^2$, where $\rho_{\mathrm{tot}}$ is the *total* mass density (stars, gas, dark matter).

3. The mass is $M(< r) = (2\mathrm{k_B}T/G\mu m_p)r = 3.1 \times 10^{12} M_\odot$ since $\mu = 0.5$ for a fully ionised hydrogen gas.

4. Writing the density as $\rho(r) = A/r^2$, and assuming $\sigma(r) = \sigma_0$ is constant, the Jeans equation becomes

$$\frac{G(M < r)}{r^2} = \frac{2\mathrm{k_B}T}{\mu m_p}\frac{1}{r} = \frac{2}{r}\,\sigma_0^2\,. \tag{1}$$

   Hence this is a solution for $\sigma_0 = 575$ km s$^{-1}$.

5. Using the previous result, the circular velocity $V_c^2 = GM(< r)/r = 2\sigma_0^2$. Then we use Eq. (6.5) in the lecture notes to find for the escape velocity $v_e$, $v_e^2 = 2V_c^2(1 + \ln(r_\star/r))$ or $v_e = 1.5 \times 10^3$km s$^{-1}$.