Chapter 13

Workshop questions

13.1 Stars & Galaxies, Workshop I

Question 1.

1. Describe the Hubble sequence.

2. Spiral galaxies have blue stars, whereas those in ellipticals are red. What does this imply for the different stellar populations?

3. Define surface brightness, and demonstrate that it is independent of distance.

4. Why can't you see the Milky Way's bulge in the optical? In which wavelength might you may able to see it? Why?

5. Give the definition of a parsec, and compute its value in metres. Use 1 AU = 149.60×10^6 km.

6. Derive the equation $m - M = 5\log(r) - 5$ that relates the apparent magnitude, m, the absolute magnitude, M, and the distance r to a star in parsecs, when obscuration is neglected.

Question 2

The absolute magnitude of the Sun in the B-band is $M_{\mathrm{B}} = +5.48$, and its luminosity $L_{\mathrm{B}} = 3.86 \times 10^{26}$ W.

1. Compute the flux from a solar type star in the Andromeda galaxy, at distance $r = 710$ kpc. Compute its apparent magnitude in the B-band.

2. Assume that the density of stars in Andromeda falls exponentially with height above the disk, $n(z) = n_0 \exp(-|z|/z_0)$, with $n_0 = 1$ pc^{-3}, where the scale-height $z_0 = 0.1$ kpc. Compute the surface density (σ) of stars (the number of stars seen per unit area), when the disk is seen face on.

3. Consider the flux, F, received from Andromeda from a square with side-length $\theta = 1$ arc seconds, assuming all its stars are like the Sun. Convert this to magnitudes. Assume Andromeda's disk is seen face-on.

4. Assume Andromeda has an exponential disk, where the number density of stars in cylindrical coordinates (R, z) follows

$$n(R, z) = n_0 \exp(-R/R_0) \exp(-|z|/z_0)\,, \tag{13.1}$$

with the disk scale-length $R_0 = 3$ kpc. Compute and plot the surface brightness in the B-band, $\mu_{\mathrm{B}}(R)$. Use the same values for n_0 and z_0 as above, and assume the disk to be face-on.

13.2 Stars & Galaxies, Workshop II

Question 1.

1. What are Cepheid variables? Explain how they can be used to estimate distances to galaxies.
2. What are *globular clusters?*
3. What are roto-vibrational transitions in molecules? What is the nature and origin of the light emitted by the dust in spiral galaxies?
4. How are the galactic coordinates (l, b) defined?
5. What is HI, HII and H_2 gas? What is CII?

Question 2. HII regions

1. A gas cloud has uniform density of $n = 5$ hydrogen atoms per cm^3. A central O stars emits $\dot{N}_\gamma = 10^{48}$ ionising photons per second. Compute the radius (location) in parsecs and speed of the ionisation front in km s^{-1} after time $t = 10^4$ yrs.
2. Compute the Strömgren radius in parsecs in this cloud. Take $\alpha = 3.1 \times 10^{-13}$ cm^3 s^{-1} as the value of the recombination coefficient.
3. Taking into account recombinations, demonstrate that the speed of the ionisation front is given by
$$\dot{N} = 4\pi R^2\, n\, \dot{R} + \frac{4\pi}{3}\, \alpha\, n^2\, R^3\,. \tag{13.2}$$
4. Show that $R(t) = R_0\,(1 - \exp(-t/t_0))^{1/3}$ is the general solution to this equation. Obtain expressions for t_0 and R_0. Show this general solutions reverts to the solution at early times (when recombinations are negligible) as well as the solution at late times (when the ionisation front stalls at the Strömgren radius).

[1 pc $= 3.09 \times 10^{16}$ m, $M_\odot = 2.0 \times 10^{30}$ kg, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}]

13.3 Stars & Galaxies, Workshop III

Question 1.

1. What is the *rotation curve* in a galaxy?

2. What are the main assumptions of the *local group timing argument*? How can we estimate the mass of the Milly Way based on the local group timing argument? Which observations are needed, and how were they obtained?

Question 2.

1. Consider a spherical density distribution $\rho(R) = \rho_0 \, (R_0/R)^\alpha$, with ρ_0, R_0 and α constants. Calculate and sketch the circular velocity as function of radius for the cases $\alpha = 0$, 1 and 2.

2. Observed rotation curves of galaxies are typically constant in their outskirts, and fall towards the center as in Figure 1. However the shape in their inner parts can differ, typically the rotation curves of massive spirals rise quickly (e.g. NGC 5533) and those of smaller (dwarf) galaxies rise much more slowly (e.g. IC 2574). Sketch the density distribution that you infer for NGC 5533, UGC 128 and IC 2574.

3. Estimate the enclosed mass in those 3 galaxies (i.e. NGC 5533, UGC 128 and IC 2574) at 10 kpc, and for the two most massive of those, at $r = 50$ kpc

4. The Milky Way hosts a super massive black hole with mass $M_{\rm BH} \sim 10^6 M_\odot$ in its centre. How close to the black hole do you expect to notice its effect on the circular orbits of stars?

5. Demonstrate that the gravitational force on a point mass due to an infinity sheet is independent of the distance to the sheet. Use this to make a simple model of how stars move perpendicular to the galactic disc.

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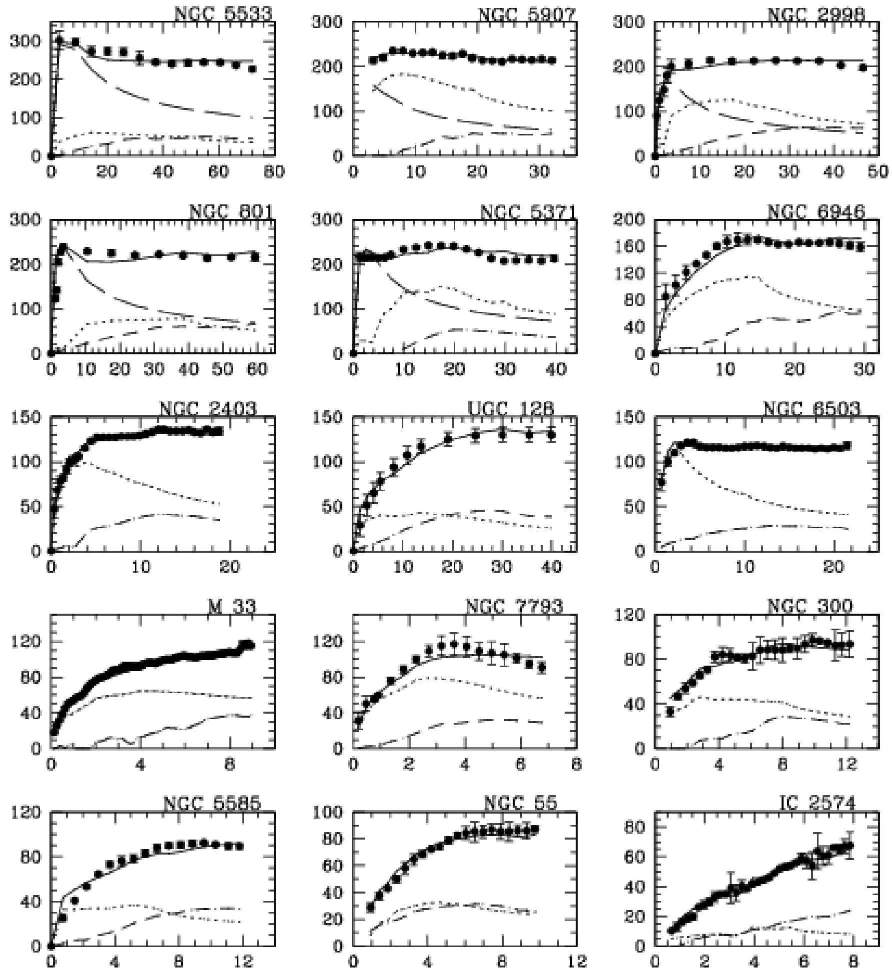

Figure 13.1: Observed rotation curves for a set of spiral galaxies.

13.4 Stars & Galaxies, Workshop IV

Question 1.

1. Which physical process produces the diffuse X-ray emission detected in ellipticals and clusters of galaxies?

2. How can the motions of galaxies in a cluster of galaxies be used to infer its mass? Why did this suggest the presence of dark matter in clusters?

3. How have clusters of galaxies been used to determine the mean dark matter density of the Universe?

Question 2. The Coma cluster of galaxies is at a distance of $d = 90$ Mpc. It contains 1000s of galaxies in a radius of $R = 3$ Mpc.

1. The observed radial velocity dispersion of galaxies in Coma is $\sigma_{1D} = 850$km s^{-1}. Use the virial theorem to estimate its total mass M.

2. Compute the virial temperature of Coma, assuming a pure hydrogen gas. Compare your answer to the binding energy of the hydrogen atom, 13.6 eV. Do you expect the gas to be mostly neutral, or mostly ionised? Use this to decide what is the appropriate mean molecular weight to use.

3. Assume that the ratio of dark matter to gas mass in the cluster, $M_{\text{dark matter}}/M_{\text{gas}} = 6$. Compute the total gas mass, and the mean electron density n_e.

4. The X-ray emissivity per unit volume, $L_x = 1.42 \times 10^{-27} n_e^2 T^{1/2}$ erg s^{-1} cm^{-3}, where n_e is the electron density in particles per cm^3, and T is the gas temperature in Kelvin. Use this to compute the total X-ray luminosity of Coma.

5. The Chandra X-ray telescope has a collecting area of $S = 800$ cm^2. Assuming all X-rays have energy 10 keV, estimate how many X-ray photons per second Chandra detects when pointing at Coma.

6. The metal abundance inferred from the X-ray spectrum is 1/3 of the solar value, $Z_\odot = 0.02$ (This is the fraction by mass of elements heavier than Helium over the total mass.) Compute the total mass in metals in Coma. The typical stellar yield, $y = M_{\rm metal}/M_{\rm star} \approx 0.02$, *i.e.* nuclear fusion in a stellar population of mass $M_\star$, will produce $y\,M_\star$ metals before the stars die. Use this to estimate the total mass that once was in stars. What fraction of the total gas mass is this?

7. Most of these metals are produced in Super Nova (SN) explosions. There is typically 1 SN per 100 solar masses of stars. Compute the total number of SNe explosions that took place in Coma. If they occur equally spaced in time, what is the current rate? Assume $t_0 = 13$ Gyear for the age of Coma.

8. The explosion energy of a SN is $E_{\rm SN} = 10^{51}$ erg. Compare the total energy input of SN with the thermal energy of the gas. Can SNe be responsible for heating the gas to the observed temperature?

[$G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $M_\odot = 1.99 \times 10^{30}$ kg,
1pc $= 3.09 \times 10^{16}$ m, $\rm k_B = 1.38 \times 10^{-23}$ J K^{-1}, $m_{\rm h} = 1.67 \times 10^{-27}$ kg]

13.5 Stars & Galaxies, Workshop V

Question 1.

1. What is the 'density-morphology' relation of galaxies?
2. The stellar mass-to-light ratio $\Gamma = M_\star/L_\star$ depends on the properties of the stellar population of a galaxy. Is Γ higher or lower for an elliptical galaxy compared to a spiral galaxy?
3. What are the Tully-Fischer and the Faber-Jackson relations? How can these be used as standard candles?
4. What is the Eddington limit of an object? Are there any objects with luminosity higher than their Eddington limit?
5. What observational evidence do we have that the MW harbours a super massive black hole?

Question 2.

1. The observed rotation speed of a distant galaxy is $V_c = 180$ km s^{-1} and its flux is $F = 4.8 \times 10^{-11}$ W m^{-2}. Estimate the¡ distance to the galaxy in Mpc assuming it lies on the Tully-Fisher (TF) relation. Assume the Milky Way, with luminosity $L_{\rm MW} = 3 \times 10^{10} L_\odot$ and circular velocity $V_{c,\rm MW} = 220$ km s^{-1} also lies on the TF relation.
2. The Schwarzschild radius of a black hole with mass M is

$$R_S = 2GM/c^2\,. \tag{13.3}$$

   Gas on a circular orbit with radius $R = 2R_s$ emits an emission line with energy $E = 6.38$ keV - the Iron K-α line. Assume the disk is seen edge on so light on one side is Doppler shifted to the red and on the other side is shifted to the blue. Calculate the wavelength you observe this line on either side, assuming the black hole is lurking in the centre of a galaxy that moves away from us at speed $v = 12000$ km s^{-1}.
3. Figure 13.2 relates black hole mass to luminosity for a set of QSOs. Calculate the Eddington luminosity for QSO J0100+2802 (black hole

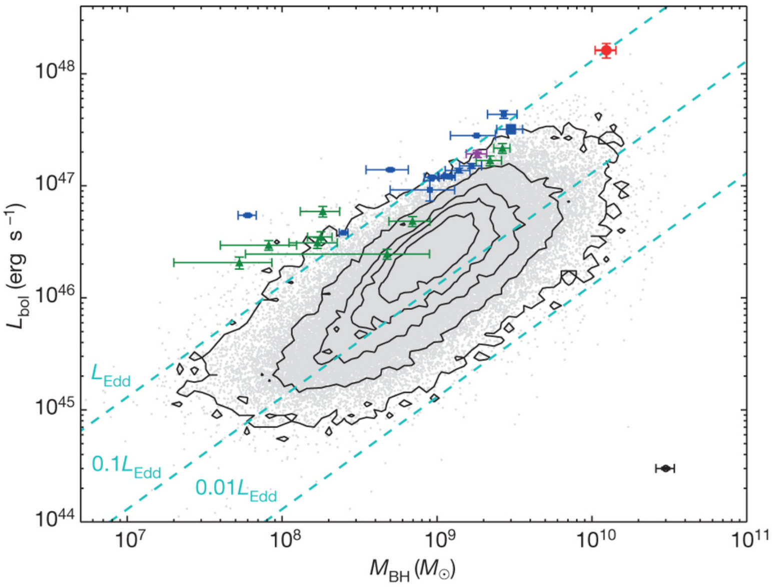

Figure 13.2: Bolometric (total) luminosity versus mass for a set of quasars from the SDSS survey (contours), a set of $z > 6$ QSOs (blue squares) and the recently discovered QSO J0100+2802, red. (from Wu et al, *Nature* 512, February 2015)

mass is $1.2 \times 10^{10} M_\odot$). This QSO is seen when the Universe was only $t = 0.884$ Gyr old. Assume the BH grew from a seed-mass of $M_{\rm seed} = 500 M_\odot$ and has been accreting at the Eddington rate ever since. Calculate when the seed formed. Assume the luminosity of the QSO depends on the accretion rate as $L = 0.1\dot{M}c^2$.

4. The presence of a point mass M distorts (curves) space-time around it so that light rays passing near M at distance b appear to be deflected by an angle.

$$\alpha = \frac{4GM}{bv^2}\,, \tag{13.4}$$

where $v = c$ is the speed of light. Show this is *twice* the angle under which a test mass moving at speed v is deflected in case the deflection angle is small. Consider a situation under which observer, lensing mass

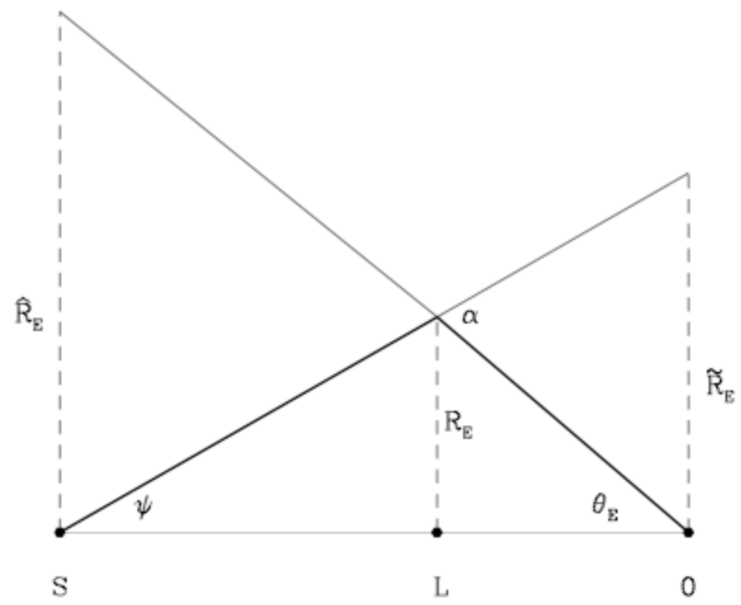

Figure 13.3: Situation where source S is gravitationally lensed by the lens L into an Einstein ring, seen by the observer O under an angle θ.

M and source are exactly aligned. Demonstrate the observer sees the source images in a ring and calculate the angle θ_E ('Einstein radius') under which the observer sees this ring (Fig. 13.3). Calculate θ_E in arc seconds, for the case where the distance observer-lens=500 Mpc, the distance lens-source=500 Mpc, and the mass of the lens $M = 10^{13} M_\odot$.

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