1 Stars & Galaxies, Galaxies example class 1, answers

Question 1.

1. Spiral: disk, old+young stars, in field vs Elliptical spheroidal, old stars, in groups and clusters. Third type: irregular galaxies: no well-defined shape, evidence for starformation and dust.

2. In the absence of absorption, the observed flux from a star with luminosity L decreases with distance r as flux $= L/(4\pi r^2)$. Since magnitude $m = -2.5\log(\text{flux}) + c$, where c is a constant, this gives $m = 5\log(r) - 2.5\log(L/4\pi) + c$. By definition, the apparent magnitude m and the absolute magnitude M are the same, when $r = 10$pc. Hence $M = 5 - 2.5\log(L/4\pi) + c$. ($r$ in pc). Substracting these two equations yields $m - M = 5\log(r) - 5$.

3. Disk (most of the light, planar, spiral arms, exponential surface brightness), Central spheroidal bulge ($\mathrm{r}^{1/4}$ law, most of the remaining light), spheroidal power-law halo, containing globular clusters.

Question 2

1. The number of stars out to radius r is $N = n(4\pi/3)r^3$ in a spherical system with constant stellar density n. If the limiting magnitude is m, then the maximum distance, r, to which we can see stars brighter than m, is given by $m - M = 5\log(r) - 5$, or $r = \mathrm{dex}[0.2(m - M + 5)]$. Hence $N = (4\pi/3)\, n\, \mathrm{dex}[0.6(m - M + 5)]$ and $\log(N) = 0.6(m - M + 5) + \log((4\pi/3)n)$, is a straight line in a $\log(N) - m$ graph, with slope 0.6

2. When there are two types of stars, absolute magnitudes $M_{1,2}$ and densities $n_{1,2}$, then we can compute as above the limiting radii $r_{1,2}$. Then, since $N = N_1 + N_2$, we get

$$\begin{aligned}\log(N_1 + N_2) &= 0.6(m - 5) + \log(4\pi/3) \\ &+ \log(n_1 \mathrm{dex}(-0.6M_1) + n_2 \mathrm{dex}(-0.6M_2))\,. \end{aligned} \quad (1)$$

This line is parallel to the line from (1), but shifted upward since there are more stars.

3. For a finite system, the relation is the same as above, out to some limiting magnitude, beyond which there are no fainter stars. The limit here is $m = 5.5 + 5\log(10^3) - 5 = 15.5$. So the cut-off is earlier than for an infinite system, *i.e.*, we have detected the edge of the system.

4. With little dust obscuration, we need to solve $m - M = 5\log(r) - 5 + A'r$ for r given m, assuming $A'r$ is small. Let $A' \equiv 5 \times A\log(e)$, and $y \equiv \mathrm{dex}(0.2(m - M + 5))$. Then $y = r\exp(A\,r) \approx r\,(1 + Ar)$. So r is the solution of $Ar^2 + r - y = 0$ for small A. The solution[1] is

$$r = \frac{-2y}{-1 - (1 + 4Ay)^{1/2}} \approx y(1 - Ay)\,. \tag{2}$$

Substituting gives

$$N = (4\pi/3)\,n\,r^3 = N_0\,(1 - 3Ay)\,, \tag{3}$$

where N_0 is without obscuration. Finally,

$$\log(N) = \log(N_0) - 0.6\,A'\mathrm{dex}[0.2\,(m - M + 5)]\,. \tag{4}$$

The number of stars drops below the no-obscuration line $\log(N) = \log(N_0)$ by $-0.6a\,A'\mathrm{dex}[0.2\,(m - M + 5)]$.

[1] The general solution to $ax^2 + bx + c = 0$ is $x = (-b \pm (b^2 - 4ac)^{1/2})/2a$, but another is $x = 2c/(-b \pm (b^2 - 4ac)^{1/2})$