Stars & galaxies, example class 2

Question 1.

1. Rotation curve is the circular velocity as function of radius. On a circular orbit, centripetal acceleration balances gravity: $GM(< r)/r^2 = V_c^2/r$. For a Keplerian disk, M is constant, hence $V_c = (GM/r)^{1/2}$.

2. Light is concentrated in the centre, whereas dynamical evience suggests there to be a lot of mass in the outer parts of the MW. This evidence is from (1) flat rotation curve as inferred from Oort's constants or HI 21-cm measurements, (2) evidence from high velocity stars, (3) motion of Andromeda. (Any 2 will do)

3. In a differentially rotating disk, a spiral pattern will quickly wind-up, because the orbital period of stars with small radii is much smaller than the period of stars with large radii, but observed spiral arms patterns are not all tightly wound.

Question 2.

1. Solving $GM(< r)/r^2 = V_c^2/r$ for $M(< r)$ yields $M(< r) = V_c^2 r/G$. Taking the derivative wrt r gives $dM(< r)/dr = V_c^2/G$ when V_c is constant. Substituting $dM(< r)/dr = 4\pi r^2\rho(r)$ shows that $\rho(r) = V_c^2/(4\pi G r^2)$.

2. Substituting V_c=220km s^{-1}, and $r = 8,5$kpc yields $\rho = 8.4 \times 10^{-25}$ g cm^{-3} which is equivalent to 0.5 protons cm^{-3}.

3. The rotation velocity is

$$\begin{aligned} V_c^2 &= V_{c,\mathrm{halo}}^2 + \frac{GM_b}{r_b}\left(\frac{r}{r_b}\right)^2 \quad \text{for } r \le r_b \\ &= V_{c,\mathrm{halo}}^2 + \frac{GM_b}{r} \quad \text{for } r \ge r_b \end{aligned} \tag{1}$$

   For $V_{c,\mathrm{halo}}^2 \gg \frac{GM_b}{r}$, the rotation curve is flat, it increases toward $r = r_b$ where it has a maximum $V_c \approx 250$km s^{-1}, then decreases again.

4. The new density at the position of the Sun, taking into account the Bulge, is $\rho = 7.5 \times 10^{-25}$ g cm^{-3} which is equivalent to 0.45 protons cm^{-3}.

5. This implies $M(< r) \propto r^3$ or constant density core.

6. $P = (2\pi r/v_c) = 7.5 \times 10^{15} s = 2.4 \times 10^8$yr.