Stars & galaxies, examples class 3

Short Questions:

1. Describe two ways that have been used to estimate the mass of galaxy clusters.
2. How have galaxy clusters been used to estimate the mean dark matter density of the Universe?
3. What is the Tully-Fisher relation? How is it used as a standard candle?

Long Question: Clusters of Galaxies

The Coma cluster of galaxies is at a distance of $d = 90$ Mpc. It contains 1000s of galaxies in a radius of $R = 3$ Mpc.

1) The observed radial velocity dispersion of galaxies in Coma is $\sigma_{1D} = 850$km s^{-1}. Use the virial theorem to estimate its total mass M.

2) Compute the virial temperature of Coma, assuming a pure hydrogen gas. Compare your answer to the binding energy of the hydrogen atom, 13.6 eV. Do you expect the gas to be mostly neutral, or mostly ionised? Use this to decide what is the appropriate mean molecular weight to use.

3) Assume that the ratio of dark matter to gas mass in the cluster, $M_{\rm dark\ matter}/M_{\rm gas} = 6$. Compute the total gas mass, and the mean electron density n_e.

4) The X-ray emissivity per unit volume, $L_x = 1.42 \times 10^{-27} n_e^2 T^{1/2}$ erg s^{-1} cm^{-3}, where n_e is the electron density in particles per cm^3, and T is the gas temperature. Use this to compute the total X-ray luminosity of Coma.

5) The Chandra X-ray telescope has a collecting area of $S = 800$ cm^2. Assuming all X-rays have energy 10 keV, estimate how many X-rays photons per second Chandra detects when pointing at Coma.

6) The metal abundance inferred from the X-ray spectrum is 1/3 of the solar value, $Z_\odot = 0.02$. Compute the total mass in metals in Coma. The typical stellar yield, $y = M_{\rm metal}/M_{\rm star} \approx 0.02$, *i.e.* nuclear fusion in a stellar population of mass $M_\star$, will produce $y\, M_\star$ metals before the stars die. Use this to estimate the total mass that once was in stars. What fraction of the total gas mass is this?

7) Most of these metals are produced in Super Nova (SN) explosions. There is typically 1 SN per 100 solar masses of stars. Compute the total number of SNe explosions that took place in Coma. If they occur equally spaced in time, what is the current rate? Assume $t_0 = 13$ Gyear for the age of Coma.

8) The explosion energy of a SN is $E_{\mathrm{SN}} = 10^{51}$ erg. Compare the total energy input of SN with the thermal energy of the gas. Can SNe be responsible for heating the gas to the observed temperature?

[$G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $M_\odot = 1.99 \times 10^{30}$ kg, 1pc $= 3.09 \times 10^{16}$ m, $\mathrm{k_B} = 1.38 \times 10^{-23}$ J K^{-1}, $m_\mathrm{h} = 1.67 \times 10^{-27}$ kg]