Stars & galaxies, weekly problems

Problem 1.

The expression for the Jeans mass in a gas of temperature T and density ρ is

$$M_J = \left(\frac{3k_B T}{2m_p G}\right)^{3/2} \left(\frac{3}{4\pi\rho}\right)^{1/2} ,$$

where $\mathrm{k_B} = 1.38 \times 10^{-23} \mathrm{J}\, K^{-1}$ is Boltzmann's constant, $m_p = 1.67 \times 10^{-27}$kg is the proton mass and $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2} Newton's constant.

Show that

$$M_J = 1.5 \times 10^4\, M_\odot (n/\mathrm{cm}^{-3})^{-1/2}\, (T/100K)^{3/2} ,$$

when $n = \rho/m_p$ is the particle density, and $M_\odot = 1.99 \times 10^{30}$ kg is the solar mass. [2 marks]

Suppose two clouds with density $n = 1\mathrm{cm}^{-3}$, $T = 100$K and $M = 10^4 M_\odot$ collide. Due to the shock, the density n increases by a factor of four. Compute the maximum T below which the new cloud will be Jeans unstable. [4 marks]

Determine whether the cloud is stable if the gas is isothermal. [2 marks]

Determine whether the cloud is stable if the gas behaves adiabatically, ($T \propto \rho^{2/3}$). [2 marks]