

Stars & galaxies, weekly problems

Problem 2.

Short questions

1. What is the Hydrogen 21-cm line? How is it used to measure rotation curves of galaxies? [2 marks]
2. What are Oort's constants? Why do they suggest the presence of dark matter in the Milky Way? [2 marks]

Long question

The speed, $V_c(r)$, of a particle on a circular orbit with radius r enclosing a mass $M(< r)$ is $V_c^2 = GM(< r)/r$.

Suppose a galaxy has a density distribution $\rho(r)$ given by

$$\begin{aligned}\rho(r) &= \rho_c \text{ for } r \leq r_c \\ &= \rho_c (r_c/r)^2 \text{ for } r_c \leq r \leq r_h \\ &= 0 \text{ for } r \geq r_h,\end{aligned}$$

i.e. it has a constant density core (density ρ_c) and a $1/r^2$ halo, which cuts-off sharply at r_h . $\rho_c, r_c < r_h$ are constants.

Show that the enclosed mass, $M(< r)$, is

$$\begin{aligned}M(< r) &= M_c (r/r_c)^3 \text{ for } r \leq r_c \\ &= M_c (1 + 3 \frac{r - r_c}{r_c}) \text{ for } r_c \leq r \leq r_h \\ &= M_T \text{ for } r \geq r_h,\end{aligned}$$

where $M_c = (4\pi/3)\rho_c r_c^3$ is the mass of the core, and $M_T = M_c(1 + 3(r_h - r_c)/r_c)$ is the total mass of the galaxy. [3 marks]

Compute the circular velocity, $V_c(r)$, as function of radius r . Sketch the result. [3 marks]