Stars & galaxies, homework problems

Problem 3. Hydrostatic equilibrium of the hot gas and velocity dispersion of stars in an elliptical galaxy.

1. Explain how X-ray observations are used to determine the temperature, $T(r)$, and density, $\rho_{\rm gas}(r)$, of the hot gas in an elliptical galaxy. [2 marks]

2. X-ray analysis shows that T is constant, and $\rho_{\rm gas}(r) \propto 1/r^2$. Use the equation of hydrostatic equilibrium to show that the total mass density (stars+gas+dark matter) follows $\rho \propto 1/r^2$. [2 marks]

3. Compute the enclosed mass within a radius $r_\star = 20$kpc. Assume the gas is fully ionised hydrogen and $T = 2 \times 10^7$K. [2 marks]

4. The equation for the isotropic velocity dispersion of the stars, σ, is the Jeans equation,
$$\frac{GM(<r)}{r^2} = -\frac{1}{\rho}\frac{d}{dr}\left(\rho\sigma^2\right). \qquad (1)$$
Given the density profile found earlier, show that the velocity dispersion is constant, $\sigma(r) = \sigma_0$, and compute σ_0. Hint: show that $\sigma^2 - \sigma_0^2 = \frac{r}{2}\frac{d}{dr}(\sigma^2 - \sigma_0^2)$ with $\sigma_0^2 = (\mathrm{k_B}T/\mu m_p)$. [2 marks]

5. Find the escape velocity for this galaxy at $r = 15$kpc, assuming the radius beyond which $\rho = 0$ is $r_\star = 20$kpc. [2 marks]