Stars & galaxies, weekly problems

Problem 1.

$$\begin{aligned}
M_J &= \left(\frac{3\,\times 1.38\times 10^{-23}\times 100}{2\times\,1.67\times 10^{-27}\times 6.67\times 10^{-11}}\right)^{3/2}\left(\frac{3}{4\pi\times\,1.67\times 10^{-27}\,\times 10^{6}}\right)^{1/2} \\
&\times\ (n/\mathrm{cm}^3)^{-1/2}(T/100\mathrm{K})^{3/2}\mathrm{kg} \\
&= 1.5\times 10^4 M_\odot\,(n/\mathrm{cm}^3)^{-1/2}(T/100\mathrm{K})^{3/2}\,. \qquad (1)
\end{aligned}$$

After the shock, the mass of the cloud is twice the mass of the original cloud, $M = 2{\times}10^4 M_\odot$. The new Jeans mass is $M_J = 1.5{\times}10^4 M_\odot\,(4)^{-1/2}(T'/100\mathrm{K})^{3/2}$, where T' is the temperature after the shock. Hence $M = M_J$ for $(T'/100\mathrm{K})^{3/2} = 8/3$ or $T' = 192\mathrm{K}$.

For isothermal gas, $T = 100\mathrm{K}$ is below T_{max}, and so the cloud is Jeans unstable.

For adiabatic gas, the temperature after the shock is $T' = 4^{2/3}T = 252\mathrm{K}$, the cloud's mass is below the Jeans mass and is hence is stable.