Stars & galaxies, weekly problems

Problem 2.
Short Questions

1. The energy of hydrogen atom is higher when the spins of proton and electron are parallel vs anti-parallel. The 21-cm line corresponds to the transition to the lower-energy state. The wavelength λ of the 21-cm line is Doppler shifted, so by measuring $\lambda(r)$ one can determine the rotation curve. Optionally: add that the HI gas is on a circular orbit and so can be used to trace the rotation curve.

2. Oort's constants A and B describe how the radial and tangential velocity of stars in a differentially rotating disk, where stars are on circular orbits, depend on distance and direction to a star. The measured values of A and B differ significantly from the values expected for a Keplerian disk, in particular, the measured gradient of the circular velocity is much smaller than the value for a Keplerian disk, suggesting there is much more mass at large radii than is seen.

Long Question
In general, $M(< r) = 4\pi \int_0^r \rho(r) r^2 dr$ for a spherically symmetric density distribution.

1. For $r < r_c$, $M(< r) = 4\pi\rho_c \int_0^r r^2 dr = (4\pi/3)\rho_c r^3 = M_c(r/r_c)^3$

2. For $r_c < r < r_h$, $M(< r) = M_c + 4\pi\rho_c r_c^2 \int_{r_c}^r dr = M_c(1 + 3(r - r_c)/r_c)$a

3. For $r > r_h$, we evaluate the previous expression at $r = r_h$, so $M = M_T = M_c(1 + 3(r_h - r_c)/r_c)$.

The circular velocity $V_c = (GM(< r)/r)^{1/2}$, hence using the previous results:

1. For $r < r_c$, $V_c(r) = (\frac{GM_c}{r_c^3})^{1/2} r$, or V_c increases linearly with r

2. For $r_c < r < r_h$, $V_c(r) = \left[\frac{GM_c}{r_c}(3 - \frac{2r_c}{r})\right]^{1/2}$, which is nearly constant for $r \gg r_c$.

3. For $r > r_h$, $V_c(r) = \left[\frac{GM_T}{r}\right]^{1/2}$, which is Keplerian fall-off.