

Stars & galaxies, homework problems

Problem 3.

1. $T(r)$ is determined from modelling the line-strengths in the X-ray spectrum, $\rho(r, T)$ from the X-ray emissivity due to thermal bremsstrahlung, $\propto \rho^2 T^{1/2}$.
2. In hydrostatic equilibrium, $GM/r^2 = -(1/\rho_g)dp/dr$, where $p = (k_B T/\mu m_p)\rho_g$ is the gas pressure and ρ_g the gas density. This simplifies when T is constant to $GM = -(k_B T/\mu m_p)r^2 d \ln \rho_g/dr$. If $\rho_g \propto 1/r^2$, the rhs of this equation becomes $(2k_B T/\mu m_p)r$, or $M(< r) \propto r$. Taking the derivative of both sides wrt r shows $\rho_{\text{tot}}(r) \propto 1/r^2$, where ρ_{tot} is the *total* mass density (stars, gas, dark matter).
3. The mass is $M(< r) = (2k_B T/G\mu m_p)r = 3.1 \times 10^{12} M_\odot$ since $\mu = 0.5$ for a fully ionised hydrogen gas.
4. With $\sigma_0^2 = (k_B T/\mu m_p)$, the Jeans equation becomes

$$\sigma^2 - \sigma_0^2 = \frac{r}{2} \frac{d}{dr} (\sigma^2 - \sigma_0^2), \quad (1)$$

the general solution of which is $\sigma^2 - \sigma_0^2 = Ar^2$. We need $A = 0$ so that σ remains finite, hence $\sigma = \sigma_0 = 406 \text{ km s}^{-1}$.

5. Using the previous result, the circular velocity $V_c^2 = GM(< r)/r = 2\sigma_0^2$. Then we use Eq. (6.5) in the lecture notes to find for the escape velocity v_e , $v_e^2 = 2V_c^2(1 + \ln(r_*/r))$ or $v_e = 1300 \text{ km s}^{-1}$.